Mathematics

Term 2 2019 Year 11

Term summary

Chapter 1 – Earning and Managing Money

1a Earning an income

An amount earned based on hours worked is called a wage. A fixed amount earned regardless of the number of hours spent working is called a salary and is described as an amount P.A (per annum meaning per year)

1 year= 365 days

=12 months

= 52 weeks

= 26 fortnights

You can use these conversions to figure out how much someone gets paid per week, month, fortnight or year.

Example:

If someone makes $283 per week, how much do they make;

Per year: $\$283 \times 52 = \$14716$

Per month: $\$14716 \div 12 = \$1226.33$

1b Allowances, bonuses and loadings

Most employees work under an award that states the maximum number of normal hours they should work per day or per week. Hours worked in excess of this (including weekends, late nights or public holidays) are classified as **overtime** and are usually paid at a higher rate than usual. Two types of over time are time and a half (x1.5) and double time (x2).

Example:

A rate of $26.20 on over time is

Time and a half: $26.20 \times 1.5 = \$39.30$

Double time: $26.20 \times 2 = \$52.40$

Employees who are required to work under difficult, unpleasant, or dangerous conditions are often paid an **allowance** that is above their normal rate of pay. Allowances are also paid for uniforms, dry cleaning, travel, meals, food, tools, etc.

Example:

If a rate of pay is $25.45 but there is an allowance for $24 every day that the odd job is done, what is the pay for someone who works 4 hours in 5 days and does the odd job two days.

Normal pay: $\$25.45 \times 4 \times 5 = \$509$

Allowance: $\$24 \times 2 = \$48$

Total Pay: $\$509 + \$48 = \$557$

A **bonus** is an extra one-off payment is paid as an incentive for employees who work hard over a period of time. Employers use bonuses to encourage employees to work harder.

Example:

An employee was getting a bonus of 6% of their normal yearly pay of $58 000.

Bonus: $6\% \ of \ 58\,000 = \frac{6}{100} \times 58\,000 = \$3480$

1c Commission, Piecework and Royalties

Commission is a method of payment usually used to pay salespeople. Commission is usually paid as a percentage of the value of goods sold. Sometimes a sales person is paid a small wage called a retainer, along with their commission.

Example:

A salesperson earns a retainer of $330 plus 13% commission on sales for a week in which her sales were $1780 what is her total earnings?

Commission: $13\% \ of \ \$1780 = \frac{13}{100} \times 1780 = \$231.40$

Total earnings: $retainer + commission = 330 + 231.40 = \$561.40$

Piecework is a system of payment by which a person is paid a fixed amount for each job or task completed.

Example:

If someone is a figurine painter and is paid $4.38 per figurine. In 1 week, they paint 143 figurines. How much would they get paid that week.

Pay: $\$4.38 \times 143 = \$626.34$

A **royalty** is a payment made to a person who owns a copyright. For example, a musician who writes a song is paid a royalty for the amount of people who buy the song. Similarly, an author of a book is paid a royalty on the number of books sold. Royalties are calculated the same was as commission and piecework.

1D Government Allowances and Pensions

Some people receive government pensions or allowances. These include pensions for old age, disabilities and sole parenthood. Allowances may include Youth allowances or Job Start.

Tables:

Students and Australian apprentices personal income test

Students and Australian apprentices	If you earn between $437 and $524 your fortnightly payment reduces by	If you earn more than $524 your fortnightly payment reduces by	Your payment reduces to $0 once your income reaches the maximum of
Single, under 18 years, at home	50 cents for each dollar you earn over $437	$43.50 plus 60 cents for each dollar you earn over $524	$857.17
Single, 18 years and over, at home	50 cents for each dollar you earn over $437	$43.50 plus 60 cents for each dollar you earn over $524	$939.34
Single or couple, no dependants, away from home	50 cents for each dollar you earn over $437	$43.50 plus 60 cents for each dollar you earn over $524	$1192.34
Couple with dependants	50 cents for each dollar you earn over $437	$43.50 plus 60 cents for each dollar you earn over $524	$1265.17
Single with dependants	50 cents for each dollar you earn over $437	$43.50 plus 60 cents for each dollar you earn over $524	$1422.34

If you earn less than $437 per fortnight, your Youth Allowance payment is not affected.

Youth Allowance rates (as at July 2017)

Circumstance	Fortnightly payment
Single, no children	
Under 18, at home	$239.50
Under 18, away from home	$437.50
18 and over, at home	$288.10
18 and over, away from home	$437.50
Single with children	$573.30
Partnered, no children	$437.50
Partnered with children	$480.50

Job seekers personal income test

Job seekers	If you earn between $143 and $250 your fortnightly payment reduces by	If you earn more than $250 your fortnightly payment reduces by	Your payment reduces to $0 once your income reaches the maximum of
Single, under 18 years, at home	50 cents for each dollar you earn over $143	$53.50 plus 60 cents for each dollar you earn over $250	$566.50
Single, 18 years and over, at home	50 cents for each dollar you earn over $143	$53.50 plus 60 cents for each dollar you earn over $250	$648.67
Single or couple, no dependants, away from home	50 cents for each dollar you earn over $143	$53.50 plus 60 cents for each dollar you earn over $250	$901.67
Couple with dependants	50 cents for each dollar you earn over $143	$53.50 plus 60 cents for each dollar you earn over $250	$974.50
Single with dependants	50 cents for each dollar you earn over $143	$53.50 plus 60 cents for each dollar you earn over $250	$1131.67
Single principal carer of dependent children, granted an exemption for foster caring, home schooling, distance education or large family	50 cents for each dollar you earn over $143	$53.50 plus 60 cents for each dollar you earn over $250	$1427.67

If you earn less than $143 per fortnight, your Youth Allowance payment is not affected.

1e Deductions and net income

The total amount earned by an employee is called gross income. The net income is calculated by subtracting deductions from the gross income

$$Net\ Income = Gross\ Income - All\ Deductions$$

Example:

If someone's gross income is $865.43 and their weekly deductions are $182.80, what is their net income?

Net income: $865.43 - 182.80 = \$682.63$

Tables:

The table below shows health insurance costs (the 30% rebate has been deducted from these premiums).

NSW health insurance premiums – single

Hospital cover	Direct debit/Payroll deduction			Advance Pay		
	Weekly	Fortnightly	Monthly	Quarterly	Half-yearly	Yearly
Top hospital no excess	$25.64	$51.27	$111.09	$333.26	$666.51	$1310.00
Intermediate hospital no excess	$19.32	$38.63	$83.70	$251.10	$502.19	$996.00
Intermediate hospital excess						
Level 1: $250	$18.59	$37.18	$80.56	$241.67	$483.34	$950.00
Level 2: $500	$16.48	$32.96	$71.41	$214.24	$428.48	$845.00
Level 3: $1000	$13.21	$26.42	$57.24	$171.73	$343.46	$680.00
Basic hospital excess						
Level 1: $250	$16.21	$32.42	$70.24	$210.73	$421.46	$835.00
Level 2: $500	$14.76	$29.52	$63.96	$191.88	$383.76	$762.00
Level 3: $1000	$11.13	$22.25	$48.21	$144.63	$289.25	$575.00

Superannuation is a way of saving for retirement. Employers contribute a base rate of 9.5% of an employer's normal income.

1f Taxable Income

$$taxable\ income = total\ income - allowable\ deductions$$

Taxable income is calculated using total income and allowable deductions. Total income includes income from all sources throughout the year including wages, salaries, bonuses, interest earned, commissions and allowances. The total income may be subject to deductions that reduce the amount used to calculate the tax payable, Tax deductions may include the cost of tools for work, the cost of safety equipment, self-educational expenses, union fees, car travel expenses, uniform costs etc.

Example:

If someone's income is $36 464 and their deductions are $518 what is their taxable income?

Taxable income: $\$36464 - 518 = \$35946$

The **Medicare levy** is an extra tax that may be payable to the government. Medicare is the public hospital system available without charge for Australian's. The Medicare levy is currently calculated at 2% of taxable income.

Example:

If someone's taxable income is $35908, what is the Medicare levy payable?

Medicare levy: $\frac{2}{100} \times 35908 = \$718.16$

Government loans are for people who need financial assistance whilst studying. These loans are repayable when taxable income exceeds a threshold. The loan repayment is paid in addition to tax. The repayment amount is a percentage of a person's taxable income.

Tables:

2016–17 repayment income thresholds and rates for HELP, SSL, ABSTUDY SSL and TSL

Repayment income (RI*)	Repayment rate
Below $54869	Nil
$54869–$61119	4.0%
$61120–$67368	4.5%
$67369–$70909	5.0%
$70910–$76222	5.5%
$76223–$82550	6.0%
$82551–$86894	6.5%
$86895–$95626	7.0%
$95627–$101899	7.5%
$101900 and above	8.0%

1G calculating tax

Tables:

Taxable income	Tax on this income
0–$18200	Nil
$18201–$37000	19c for each $1 over $18200
$37001–$87000	$3572 plus 32.5c for each $1 over $37000
$87001–$180000	$19822 plus 37c for each $1 over $87000
$180001 and over	$54232 plus 45c for each $1 over $180000

The above rates do not include the 2% Medicare levy.

Chapter 5: Perimeter, Area and Volume.

5a Perimeter:

A perimeter is the distance around the outside (border) of a shape. To find the perimeter of the shape, change all lengths to the same units and add them together.

Pythagoras' Theorem:

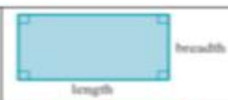
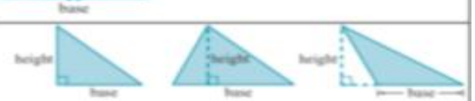
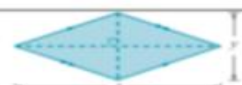
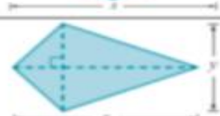
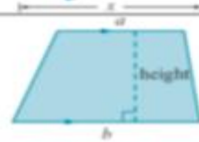
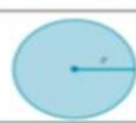
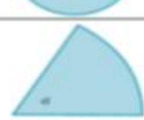
Pythagoras was a Greek Mathematician who discovered that in any right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$a^2 + b^2 = c^2$$

Circumference of a circle:

1. $pi \times diameter$
2. $2 \times pi \times radius$

5b Area of Simple Shapes

Rectangle Area = length × breadth $A = lb$	
Parallelogram Area = base × height $A = bh$	
Triangle Area = $\frac{1}{2}$ × base × height $A = \frac{1}{2}bh$	
Rhombus Area = $\frac{1}{2}$ product of the lengths of the diagonals $A = \frac{1}{2}xy$	
Kite Area = $\frac{1}{2}$ product of the lengths of the diagonals $A = \frac{1}{2}xy$	
Trapezium Area = $\frac{1}{2}$ × height × sum of the parallel sides $A = \frac{1}{2}h(a + b)$ or $A = \left(\frac{a+b}{2}\right) h$ or $A = \frac{h}{2}(a + b)$	
Circle Area = π × radius squared $A = \pi r^2$	
Sector Area = fraction of circle × area of circle $A = \frac{\theta}{360°} \times \pi r^2$	

5c Area of Composite shapes:

Composite shapes can be divided into parts, each with a simpler shape whose area's can be easily found.

Annulus:

The Shaded area in this diagram is called an annulus. An annulus is the area between two circles with the same centre but different radii. Circles with the same centre are concentric circles. The formula for the area of an annulus is

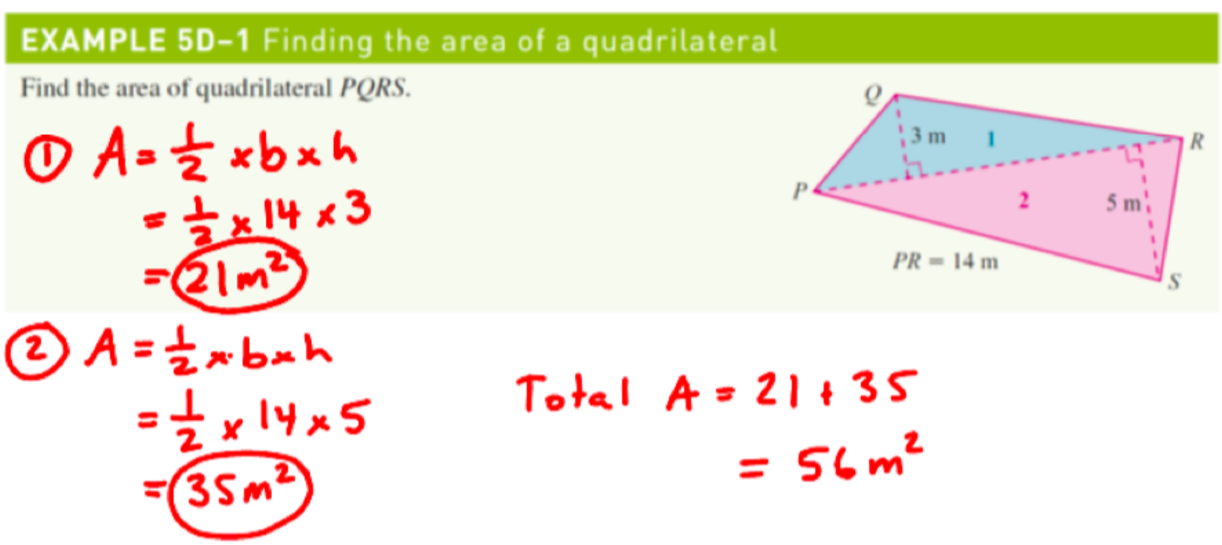
$$A = \pi R^2 - \pi r^2 \; or \; A = \pi\,(R^2 - r^2)$$

Where R is the radius of the larger circle, and r is the radius of the small circle.

5d Perimeter and area of irregular shapes

Irregularly shaped blocks can be dissected into familiar shapes for which formulas can be used.

Example:



Trapezoidal rule:

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$

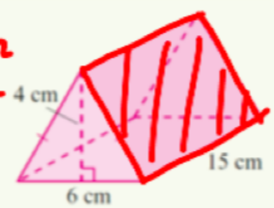
5e Surface area of a prism

A prism is a solid which has two parallel end faces that are identical. It also has a uniform cross-section when a slice is taken parallel to its end faces. To find the surface area of a prism, find the area of all the sides then add them together.

Example:

EXAMPLE 5E–3 Calculating the surface area of a triangular prism

Calculate the surface area of this triangular prism.



Triangles: $2 \times \frac{1}{2} \times 6 \times 4 = 24 cm^2$

Base: $15 \times 6 = 90 cm^2$

Pythagoras: $4^2 + 3^2 = 25$

$\sqrt{25} = 5$.

Sides: $2 \times 15 \times 5 = 150 cm^2$

$TSA = 90 + 24 + 150 = 264 cm^2$

5f Surface area of cylinders and spheres

The surface area of a cylinder of radius r and height h, open at both ends, is given by:

$$A = 2\pi rh$$

The surface area of a cylinder, of radius r and h height, closed at both ends, is given by:

$$A = 2\pi r^2 + 2\pi rh$$

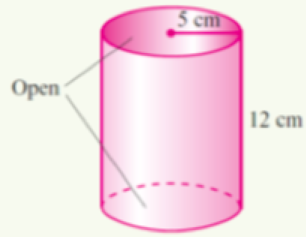
The surface area of a sphere of radius r is given by:

$$A = 4\pi r^2$$

Example:

EXAMPLE 5F–1 Finding the surface area of an open cylinder

Find the outer surface area of this open cylinder, correct to one decimal place.



$A = 2\pi rh$

$= 2 \times \pi \times 5 \times 12$

$= 376.99\ldots$

$= 377.0\text{ cm}^2$

5g Volume

Volume is the amount of space occupied by a three-dimensional object.

When a solid has a uniform cross-section, the volume of the solid can be determined using:

$$Volume = area\ of\ face \times height$$

The volume of a prism is given by:

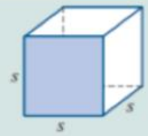
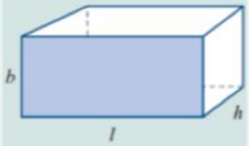
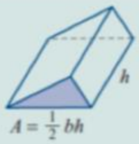
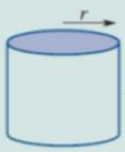
$$V = Ah$$

The volume of a cylinder is given by:

$$V = \pi r^2 h$$

The volume of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

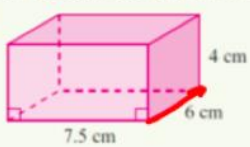
Name	Solid	Volume
Cube		$V = Ah$ $= (s^2) \times s$ $= s^3$
Rectangular prism		$V = Ah$ $= lb \times h$ $= lbh$
Triangular prism		$V = Ah$ $= \left(\frac{1}{2}bh\right) \times h$
Cylinder		$V = Ah$ $= (\pi r^2) \times h$ $= \pi r^2 h$

Examples:

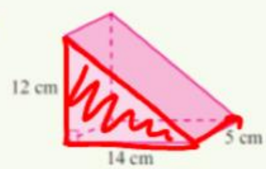
EXAMPLE 5G-2 Finding the volume of different solids

Find the volume of each of these solids.

a



b



$V = 4 \times 7.5 \times 6$

$= 180 cm^3$

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 14 \times 12$

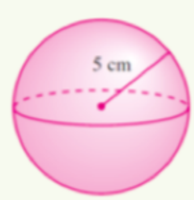
$= 84 cm^2$

$V = 84 \times 5$

$= 420 cm^3$

EXAMPLE 5G-4 Finding the volume of a sphere

Find the volume of this sphere, correct to one decimal place.



$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 5^3$

$= 523.6 cm^3$

5h Capacity

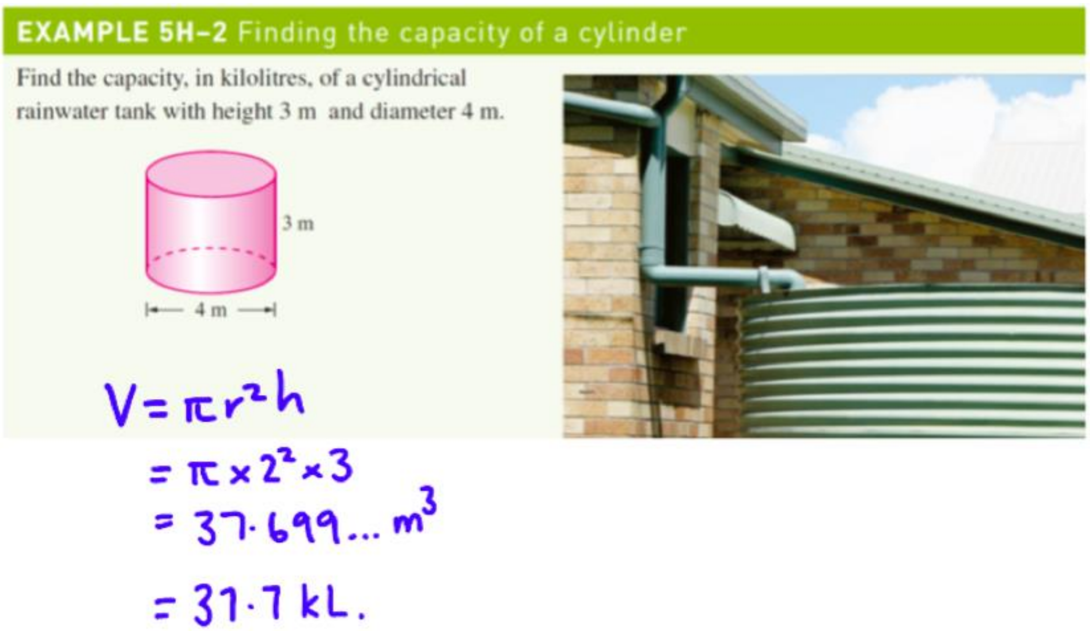
The capacity of a container is the amount of liquid it can hold. Capacity and volume are related by the following conversions

1 mL = 1 cm(cubed)

1 L = 1000 mL = 1000 cm (cubed)

1 kL = 1000 L = 1 m(cubed)

Example:



Chapter 12 Linear Relationships

12a Straight-line graphs

A linear function makes a straight-line graph. To draw linear equations, we can plot points on a number plane and draw a straight line in between them. There are two variables in a linear equation: the dependent variable and the independent variable. The value of the dependent variable relies on the value of the independent variable.

Independent variable: represented on the horizontal axis.

Dependent variable: represented on the vertical axis.

Example:

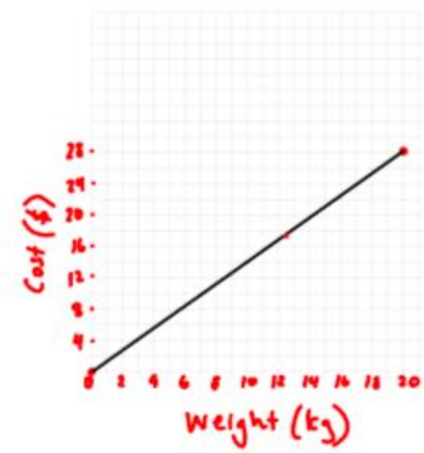
EXAMPLE 12A-1 Drawing a straight-line graph from a table of values

Bulk washing powder is sold for $1.40 per kilogram. The table shows weight versus cost for washing powder.

I	Weight (kg)	1	2	5	10	15	20
D	Cost ($)	1.40	2.80	7.00	14.00	21.00	28.00

a Draw the graph of weight versus cost.

b Use the graph to find the cost of 12.5 kg of washing powder. $18.



12b Gradient and vertical intercept

$$Gradient(slope) = \frac{vertical\ rise}{Horizontal\ run}$$

Vertical intercept of where the line crosses the y-intercept

12c The Equation y= mx+c

The conversion is to write the rule for straight-line graphs in the form $y = mx + c$ where:

X is the independent variable, because any value may be used.

Y is the dependent variable, because it depends on the value of x

M is the gradient of the line

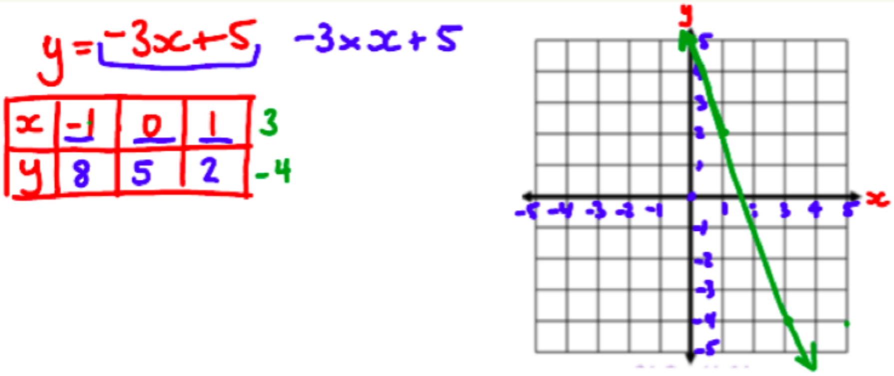
C is the y-intercept

This relationship connects every point on a straight-line graph

Example:

EXAMPLE 12C–1 Drawing a linear graph by first completing a table of values

Draw the graph of $y = -3x + 5$ by first completing a table of values.



12D Direct Variation relationships

A direct variation relationship between two quantities or variables exists when one of the quantities can be expressed as a number multiplied by the other quantity. So, if two things are directly proportional, then an increase in one causes a decrease in the other and vice versa.

In the table below, when x is multiplied by 3, y is also multiplied by 3. As one quantity increases, the second quantity increases.

× 3

x	1	2	3	4	5
y	5	10	15	20	25

× 3

In the table below, when x is divided by 4 (or multiplied by $\frac{1}{4}$) so is y. As one quantity decreases, the second quantity also decreases.

÷ 4

x	1	2	3	4	5
y	7	14	21	28	35

÷ 4

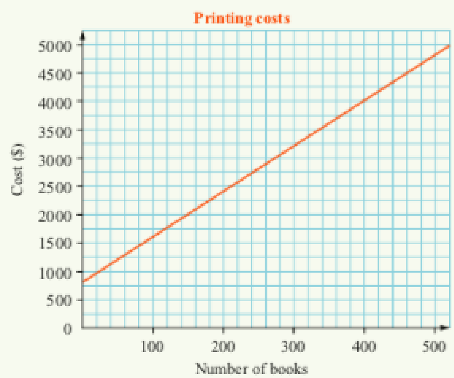
12e Linear modelling

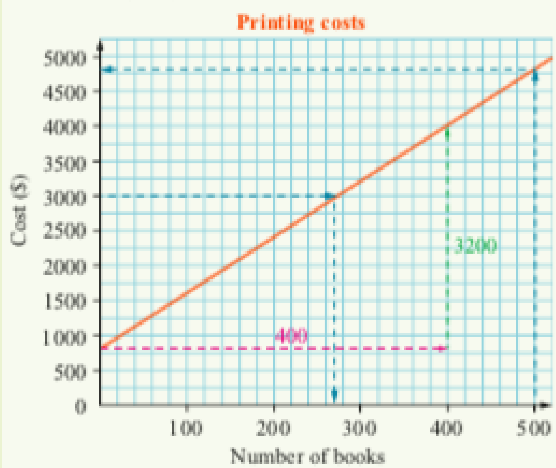
Graphs and equations can be used to model real situations. The gradient and the y-intercept of linear graphs have meaning in practical situations. A straight-line graph of the form y = mx shows a direct variation relationship between the two variables. A straight-line graph of the form y = mx + c shows a relationship between x and y, the gradient m and the constant term c.

Example:

The graph on the right models printing costs.

- **a** Find the cost of printing 500 books.
- **b** Find the number of books that can be printed for $3000.
- **c** Find the gradient of the line. What is its meaning in this situation?
- **d** Find the vertical intercept. What is its meaning in this situation?



Solve	Think	Apply
$4800	Find the point on the line whose horizontal coordinate (number of books) is 500. The vertical coordinate (cost) is $4800. 	Find the point on the line with the given coordinate and read off the unknown coordinate on the appropriate axis.
270 books	Find the point on the line whose vertical coordinate (cost) is 3000. The horizontal coordinate (number of books) is 270.	
Gradient $= \frac{3200}{400} = 8$ This is the cost per book after the initial set up costs. Cost = $8/book	Using points (0, 800) and (400, 4000): Gradient $= \frac{\text{rise}}{\text{run}} = \frac{3200}{400}$. Because the units of the vertical rise are dollars and the units of the horizontal run are books, the unit for the gradient is $/book.	The gradient is expressed as the unit of the rise per unit of the run.
Vertical intercept is $800. This is the fixed cost for setting up.	The vertical intercept is the cost of printing zero books.	The vertical intercept is the point where the line cuts the vertical axis.