

Section One: Multiple Choice

5 marks

Attempt Questions 1-5

Allow about 10 minutes for this section

Write your answer on the paper provided.

1. What are the solutions to $3x^2 - 7x - 1 = 0$?

(A) $x = \frac{-7 \pm \sqrt{37}}{6}$

(B) $x = \frac{-7 \pm \sqrt{61}}{6}$

(C) $x = \frac{7 \pm \sqrt{37}}{6}$

(D) $x = \frac{7 \pm \sqrt{61}}{6}$

2. Which of the following is irrational?

(A) $\sqrt{289}$

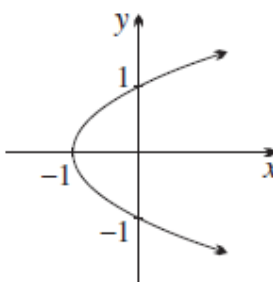
(B) $\sqrt[3]{27}$

(C) $\sqrt{11}$

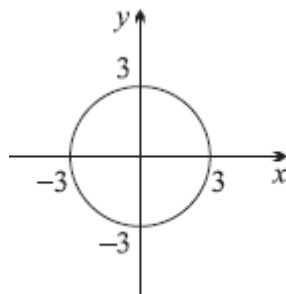
(D) $\frac{3}{7}$

3. Which of the following is function?

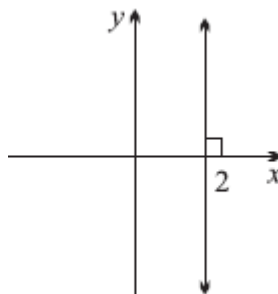
(A)



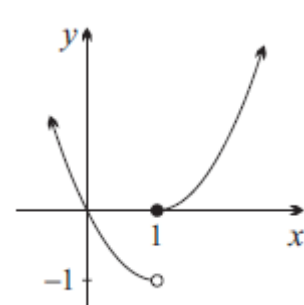
(B)



(C)



(D)



4. Which of the following is true about the function $f(x) = 2^x + 2^{-x}$

(A) Even function

(B) Odd function

(C) Zero function

(D) Neither odd or even function.

5. Simplify $\sin \theta \sec \theta$.
- (A) $\operatorname{cosec} \theta$
- (B) $\sin^2 \theta$
- (C) $\tan \theta$
- (D) $\cot \theta$.

Section Two: Extended Response

Attempt Questions 1-5

Write your answer on the paper provided.

Show all working.

Question One: Methods in Algebra (13 Marks)

Marks

- a) Expand and simplify: $3(2x-1) - 3(x+5)$. **2**
- b) Fully factorise: $x^3 + 5x^2 - 9x - 45$. **2**
- c) Express $\frac{2x-3}{6} - \frac{x-7}{15}$ as a single algebraic fraction. **3**
- d) Solve the equation: $\frac{x-2}{x+2} = 2-x$. **3**
- e) Fully simplify $\frac{x^3 - 64}{4-x} \div \frac{x^2 + 4x + 16}{5}$. **3**

Question Two: Numbers and Surds (13 Marks)**Marks****Start a new page**

a) Fully simplify the following expressions:

i) $\sqrt{27} \times \frac{1}{3}\sqrt{3}$ 2

ii) $(2\sqrt{5} + \sqrt{3})(7\sqrt{3} + \sqrt{5})$ 2

iii) $\sqrt{x^3} + \sqrt{x} - \sqrt{4x}$ 2

b) Express $0.\dot{2}\dot{7}$ as a fraction in its simplest form without the use of a calculator. 2

c) Simplify the following by rationalising the denominator $\frac{\sqrt{5}}{4\sqrt{5} + 7}$. 3

d) Given that a, b, c and d are integers, with b and d non-zero, 2
simplify the average of $\frac{a}{b}$ and $\frac{c}{d}$.

Question Three: Functions and Graphs (13 Marks)

Mark

Start a new page

- a) If $f(x) = x^2 - 3x - 40$ find $f(5) + f(-5)$ **2**
- b) Draw separate neat sketches, labelling significant features in the following:
- i) $xy = 3$ **2**
- ii) $y = x^2 + 3x - 10$ **2**
- c) State the domain and range for b) i) and b) ii). **4**
- d) A function is defined by the following: **3**
- $$f(x) = \begin{cases} \sqrt{9-x^2} & \text{for } -3 < x < 3 \\ x & \text{for } x \geq 3. \end{cases}$$

Copy and complete the table of values for $f(x)$, then sketch its graph.

x	-4	-3		-2	-1	0	1	2		3	4
y											

Question Four: Graphs and Inequations (13 Marks)**Marks****Start a new page**

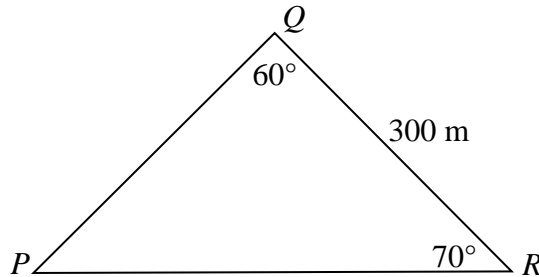
- a) Evaluate $\left|(-3)^2 - 7 \times 2\right|$. **1**
- b) Solve for x given the inequation $19 < 7x + 5 < 75$. **2**
- c) Solve $|2x + 1| \leq 9$. **2**
- d) Solve the inequality $x(x+7)(5-x) < 0$. **3**
- e) On a number plane, graph the following regions: **5**
 $(x-1)^2 + y^2 \leq 16$ and $x - y + 3 \geq 0$

And shade the intersection of the regions.

Question Five: Trigonometry (13 Marks)**Marks****Start a new page**

a) If $0^\circ \leq \theta \leq 360^\circ$ find the values of θ given $\sec \theta = \frac{2}{\sqrt{3}}$ **3**

b) In the diagram below, use the Sine rule to find the length of PQ to the nearest metre. **3**



NOT TO SCALE

c) Given that the sides of a non right angled triangle are 6 cm, 11 cm and 15 cm:

i) Show that the smallest angle in the triangle is 20° to the nearest degrees. **2**

ii) Find the area of the triangle, give your answer to 1 decimal place. **2**

d) Prove that $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos^2 \theta - \sin^2 \theta$. **3**

END OF EXAMINATION

Suggested Solutions, Marking Scheme and Markers' comments

<u>Suggested solution(s)</u>	<u>comments</u>
<p>Year 11 - 2 unit Mathematics Half Yearly Examination solutions 2015.</p> <p><u>Multiple Choice</u></p> <p>Q1. $a = 3$ $b = -7$ $c = -1$</p> $x = \frac{-(-7) \pm \sqrt{(-7)^2 - (4 \times 3 \times -1)}}{2 \times 3}$ $x = \frac{7 \pm \sqrt{49 - -12}}{6}$ $x = \frac{7 \pm \sqrt{61}}{6} \quad (D)$ <p>Q2. Rational numbers can be expressed as a fraction.</p> $\sqrt{289} = \frac{17}{1} \quad \sqrt[3]{27} = \frac{3}{1}$ <p>$\therefore \sqrt{11}$ is the only value which can not be expressed as a fraction. (C)</p> <p>Q3 Use the vertical line test.</p> <p>A function has one y value for every x value. (D)</p>	<p>Take Care with negative numbers</p>

Suggested Solutions, Marking Scheme and Markers' comments

<u>Suggested solution(s)</u>	<u>comments</u>
<p>Q4. $f(-x) = 2^{-x} + 2^{-(-x)}$ $= 2^{-x} + 2^x$ $= f(x)$</p> <p>When $f(x) = f(-x)$ it is an even function (A)</p> <p>Q5 $\sin \theta \times \frac{1}{\cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta$ (C)</p>	<p>The multiple choice questions were generally answered very well by students.</p>
<p>Extended Response Question One.</p> <p>a) $3(2x-1) - 3(x+5)$ $= 6x - 3 - 3x - 15$ ① $= 3x - 18$ ①</p> <p>b) $x^3 + 5x^2 - 9x - 45$ $x^2(x+5) - 9(x+5)$ $(x^2-9)(x+5)$ ① $(x+3)(x-3)(x+5)$ ①</p>	<p>Question One is Reference to Ch1 of textbook.</p> <p>Recognise Grouping and difference of 2 Squares.</p>

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>c) $\frac{2x-3}{6} - \frac{x-7}{15}$</p> <p>= $\frac{5(2x-3) - 2(x-7)}{30}$ ①</p> <p>= $\frac{10x-15-2x+14}{30}$ ①</p> <p>= $\frac{8x-1}{30}$ ①</p>	<p>Some students failed to successfully change the fractions to have a LCD of 30.</p> <p>'ect' marks awarded if correct simplifying after failed attempt at changing fractions.</p>
<p>d) $\frac{x-2}{x+2} = \frac{2-x}{1}$ $x \neq -2$</p> <p>$x-2 = (2-x)(x+2)$ ①</p> <p>$x-2 = 4-x^2$</p> <p>$x^2+x-6=0$ ①</p> <p>$(x+3)(x-2)=0$</p> <p>$x=-3$ $x=2$ ①</p>	<p>Recognise $(2-x)(x+2)$ = $(2-x)(2+x)$ Difference of 2 squares.</p> <p>some students struggled to get to this step by eliminating the $x+2$ denominator from the equation.</p>
<p>e) ① $\frac{(x-4)(x^2+4x+16)}{(4-x)} \div \frac{(x^2+4x+16)}{5}$</p> <p>= $\frac{(x-4)(x^2+4x+16)}{(4-x)} \times \frac{5}{(x^2+4x+16)}$ ①</p> <p>= -1×5</p> <p>= -5 ①</p>	<p>since $(4-x) = -1(x-4)$ Some students forgot diff. 2 cubes factorisation.</p>

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)

comments

Extended Response Question Two.

$$\begin{aligned} \text{a) (i)} \quad & \sqrt{27} \times \frac{1}{3}\sqrt{3} \\ & = 3\sqrt{3} \times \frac{1}{3}\sqrt{3} \quad \textcircled{1} \end{aligned}$$

$$= 3 \quad \textcircled{1}$$

$$\text{(ii) F. 0. 1. L}$$

$$14\sqrt{15} + 10 + 21 + \sqrt{15} \quad \textcircled{1}$$

$$15\sqrt{15} + 31 \quad \textcircled{1}$$

$$\text{(iii)} \quad x\sqrt{x} + \sqrt{x} - 2\sqrt{x} \quad \textcircled{1}$$

$$= \sqrt{x}(x-1) \quad \textcircled{1}$$

$$\text{b) Let } x = 0.277777 \dots$$

$$10x = 2.777777 \dots$$

$$100x = 27.777777 \dots \quad \textcircled{1}$$

$$90x = 25$$

$$x = \frac{25}{90} = \frac{5}{18} \quad \textcircled{1}$$

$$\text{c) } \frac{\sqrt{5}}{4\sqrt{5}+7} \times \frac{4\sqrt{5}-7}{4\sqrt{5}-7} \quad \textcircled{1}$$

$$\frac{20-7\sqrt{5}}{80-49} \quad \textcircled{1} = \frac{20-7\sqrt{5}}{31} \quad \textcircled{1}$$

ok ✓

generally ok ✓

many didn't factorise

Some use their calculator! @wats!

Some x by $4\sqrt{5}+7$ and some left out +

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)

comments

$$d) \frac{\frac{a}{b} + \frac{c}{d}}{2}$$

$$\frac{ad+bc}{bd} \div 2$$

$$\frac{ad+bc}{2bd}$$

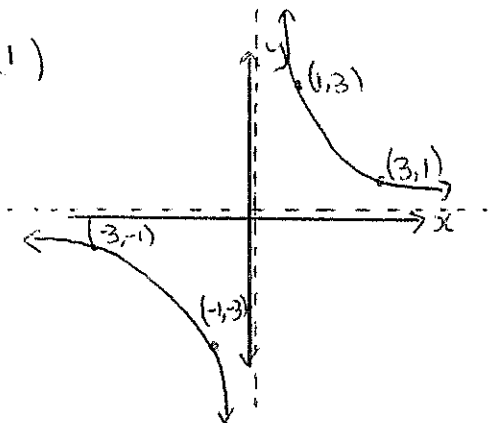
Extend Response Question 3.

$$a) f(s) = s^2 - 3s - 40 = -30$$

$$f(-s) = (-s)^2 - 3(-s) - 40 = 0 \quad \textcircled{1}$$

$$f(s) + f(-s) = -30 \quad \textcircled{1}$$

b) (1)



Many didn't attempt this question.

✓

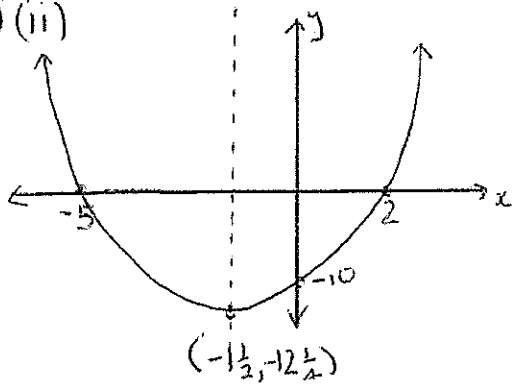
many omitted asymptotes
• very poor sketch

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)

comments

b) (ii)



$$y = (x+5)(x-2)$$

(2)

- poor sketch
- need a single smooth curve
- let for not shown

c) b(i) domain: all real x except when $x=0$ (1)

range: all real y except when $y=0$ (1)

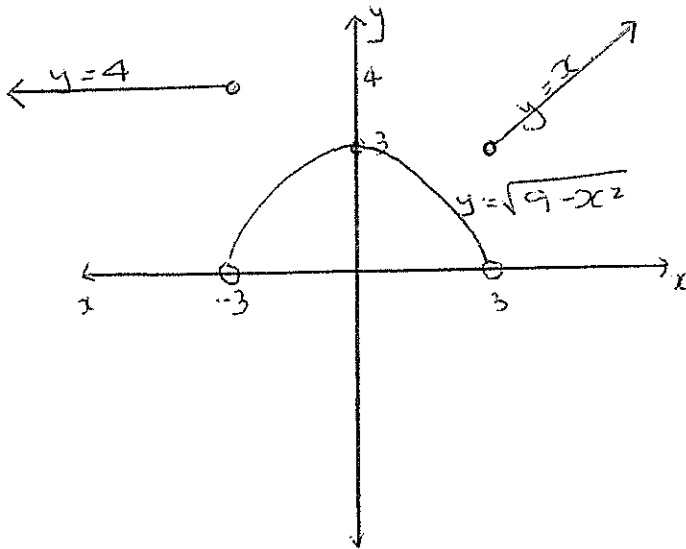
omitted $x \neq 0$
 $y \neq 0$

b(ii) domain: all real x (1)

range: all real $y \geq -12 \frac{1}{4}$ (1)

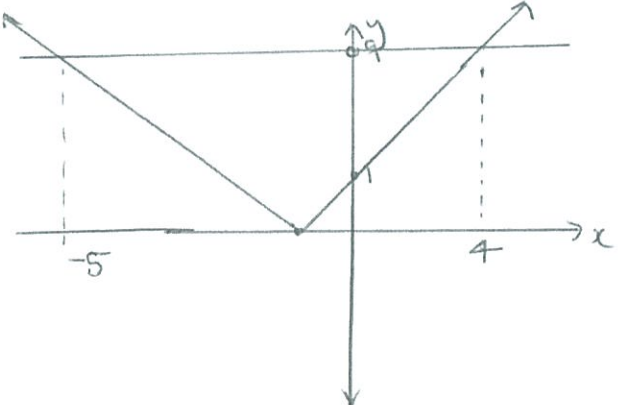
d)

x	-4	-3	-2	-1	0	1	2	3	4
y	4	4	$\sqrt{5}$	$\sqrt{8}$	3	$\sqrt{8}$	$\sqrt{5}$	3	4

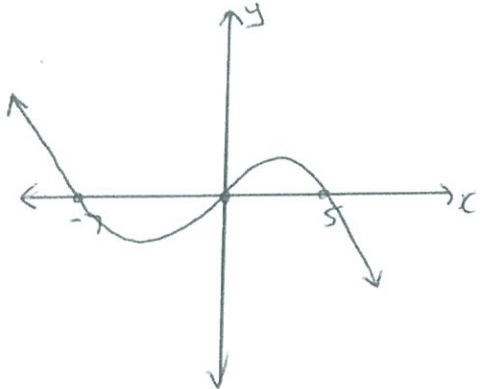
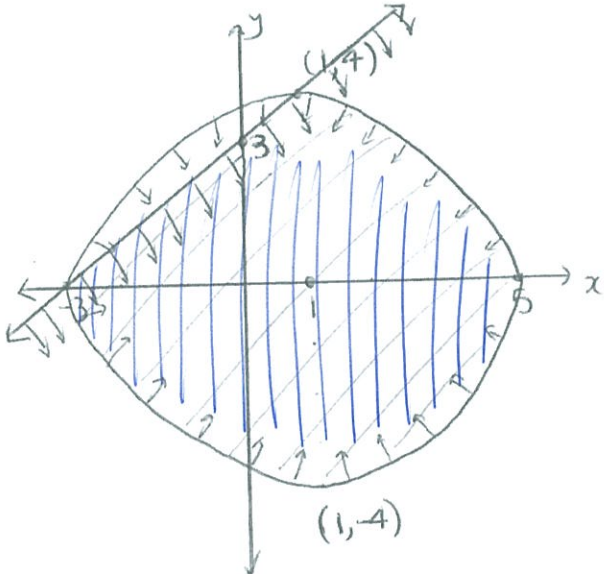


very poorly drawn.

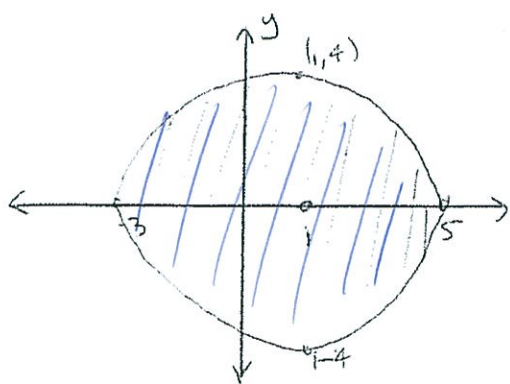
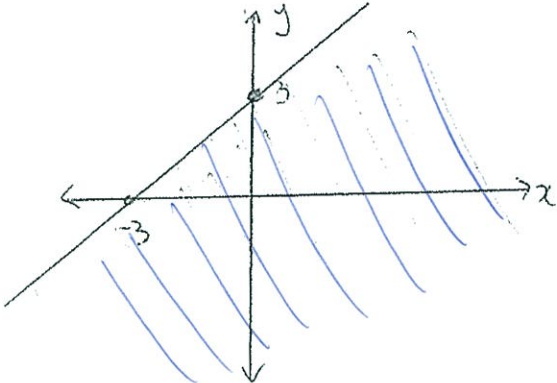
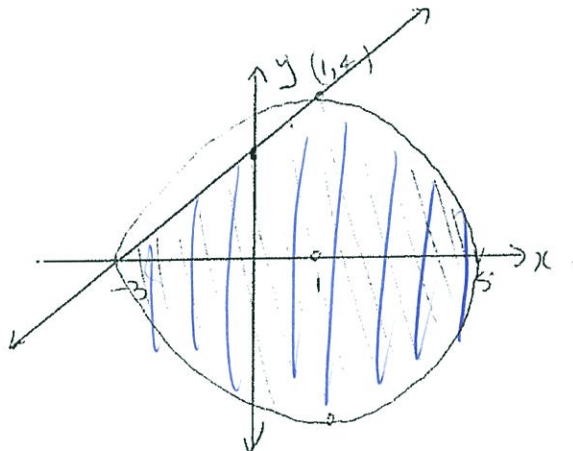
Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
Extended Response Question Four	
a) $ 9-14 = -5 = 5$ ①	only one answer = 5
b) $19 < 7x+5 < 75$ $14 < 7x < 70$ ① subtract 5 $2 < x < 10$ ① divide 7	$ 5 $ and -5 was NOT correct and written in addition to 5 meant no mark.
c) $ 2x+1 \leq 9$ $2x+1 \leq 9$ $2x+1 \geq -9$ $2x \leq 8$ $2x \geq -10$ $x \leq 4$ $x \geq -5$ ① $-5 \leq x \leq 4$ ①	often only one solution was given.
check solution makes sense. 	

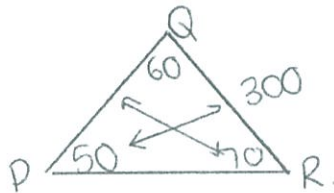
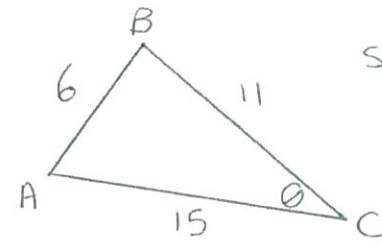
Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>d) $x(x+7)(5-x) < 0$</p> <p>zeroes occur when $x=0$ $x=-7$ $x=5$</p> <p>When $x \rightarrow \infty$ $y \rightarrow -\infty$</p>  <p>$-7 < x < 0$ $x > 5$</p> <p style="text-align: right;">below y-axis</p>	<p>1 mark for (3) value $x=0$ $x=-7$ $x=5$</p> <p>1 mark for graph/otw. If graphed incorrectly e. c. f. was awarded for final mark.</p>
<p>e)</p>  <p>2 marks for graph of circle and correct region. 2 marks for graph of line and correct region 1 mark for correct intersection.</p>	<p>Many students did not test a point and shaded the incorrect region (see over page)</p>

Suggested Solutions, Marking Scheme and Markers' comments

<u>Suggested solution(s)</u>	<u>comments</u>
<p>(e)</p> 	<p>(2)</p>
	<p>(2)</p>
	<p>(1)</p>

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>Question Five .</p> <p>a) $\frac{1}{\cos \theta} = \sec \theta$</p> $\therefore \cos \theta = \frac{\sqrt{3}}{2} \quad (1)$ $\theta = 30 \quad (1) \qquad \theta = 330 \quad (1)$	<p>Good .</p>
<p>b)</p>  <p>$\angle QPR = 50^\circ \quad (1)$</p> $\frac{PQ}{\sin 70} = \frac{300}{\sin 50}$ $PQ = \frac{300 \sin 70}{\sin 50} = 368 \text{ m}$	<p>Must use Sine Rule .</p>
<p>c)</p>  <p>smallest angle is opposite smallest side</p> $\cos \theta = \frac{15^2 + 11^2 - 6^2}{2 \times 15 \times 11} \quad (1) \qquad \cos^{-1}(\text{ANS})$ <p>(shift) $\cos \theta = 20^\circ \quad (1)$</p>	

Suggested Solutions, Marking Scheme and Markers' comments

<u>Suggested solution(s)</u>	<u>comments</u>
<p>C (ii) Area = $\frac{1}{2} ab \sin C$</p> <p>Area = $\frac{1}{2} \times 11 \times 15 \times \sin 20$</p> <p>Area = 28.2 cm^2</p> <p>d) LHS = $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$</p> <p>since $\tan^2 \theta + 1 = \sec^2 \theta$</p> <p>LHS = $\frac{1 - \tan^2 \theta}{\sec^2 \theta}$ ①</p> <p>= $\frac{1}{\sec^2 \theta} - \frac{\tan^2 \theta}{\sec^2 \theta}$</p> <p>= $\cos^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta} \div \sec^2 \theta$</p> <p>= $\cos^2 \theta - \frac{\sin^2 \theta}{\cos^4 \theta} \times \cos^2 \theta$</p> <p>= $\cos^2 \theta - \sin^2 \theta$</p> <p>= RHS.</p> <p>$\therefore$ LHS = RHS.</p> <p>$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos^2 \theta - \sin^2 \theta$</p>	<p>There is No Right Angle in the triangle.</p> <p>use LHS = RHS</p>