Centre Number | 1 | 2 | 5 |

Student Number | | | | | | | | |

CRANBROOK SCHOOL

2017

Preliminary Examination

Assessment Task 3

Mathematics

Reading time	5 minutes
Writing time	2 hours
Total Marks	70
Task weighting	50%

General Instructions

- Write using blue or black pen
- Diagrams drawn using pencil
- A Board-approved calculator may be used
- The Reference sheet is on the last page of this booklet
- Use the Multiple-Choice Answer Sheet provided
- All relevant working should be shown for each question

Additional Materials Needed

- Multiple Choice Answer Sheet
- 4 writing booklets

Structure & Suggested Time Spent

Section I

Multiple Choice Questions

- Answer Q1 – 10 on the multiple choice answer sheet
- Allow 15 minutes for this section

Section II

Extended response Questions

- Attempt all questions in this section in a separate writing booklet
- Allow about 1 hour 45 minutes for this section

This paper must not be removed from the examination room

Disclaimer

The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.

Section I

10 Marks

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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1 $\dfrac{4}{4-2\sqrt{2}}$ is equivalent to:

(A) $2+\sqrt{2}$

(B) $2-\sqrt{2}$

(C) $\sqrt{2}$

(D) $\dfrac{4+2\sqrt{2}}{3}$

2 What is the solution of $x^2-5x+6<0$?

(A) $x=2,\ 3$

(B) $x<2,\ x>3$

(C) $2<x<3$

(D) No solution

3 What is the equation of a quadratic with roots $1-\sqrt{2}$ and $1+\sqrt{2}$?

(A) $x^2+\sqrt{2}x-1=0$

(B) $x^2-2x-1=0$

(C) $x^2+2x-1=0$

(D) $x^2-\sqrt{2}x-1=0$

4 What is the gradient of a line that is perpendicular to $4x-3y+1=0$?

(A) 4

(B) -4

(C) $\frac{4}{3}$

(D) $-\frac{3}{4}$

5 What is the derivative of $(1-2x)^5$

(A) $5(1-2x)^4$

(B) $-5(1-2x)^4$

(C) $10(1-2x)^4$

(D) $-10(1-2x)^4$

6 What is the equation of a parabola with focus $(3, 5)$ and directrix $x=9$?

(A) $y^2-10y-12x+97=0$

(B) $y^2-10y+12x-47=0$

(C) $x^2-6y-12y+69=0$

(D) $x^2-6y+12y-51=0$

7 What is the coordinates of the focus in $2x^2 = 3y$

(A) $\left(\frac{8}{3}, 0\right)$

(B) $\left(\frac{3}{8}, 0\right)$

(C) $\left(0, \frac{8}{3}\right)$

(D) $\left(0, \frac{3}{8}\right)$

8 How many solutions does $2\cos^2\theta = 1$ have for $-180^\circ \le \theta \le 180^\circ$?

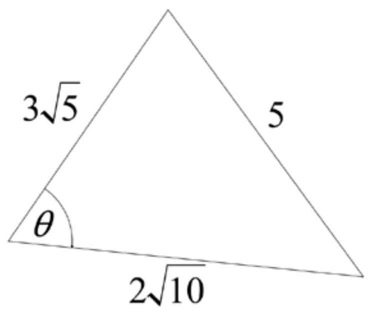
(A) 1

(B) 2

(C) 3

(D) 4

9 What is the exact value of θ in the following diagram?



(A) 30°

(B) 45°

(C) 60°

(D) 90°

10 What is the range of the function $f(x)$ defined by $f(x)=\begin{cases} -1, & x<-2 \\ x^2-4, & -2\le x<0 \\ x-4, & 0\le x\le 4 \\ 0, & x>4 \end{cases}$?

(A) All real y

(B) $y\ge -4$

(C) $y\le 0$

(D) $-4\le y\le 0$

END OF SECTION I

Section II

60 Marks

Allow about 1 hour 45 minutes minutes for this section

Answer question 11-14 in separate booklets.

Question 11 **Start a new booklet** **15 Marks**

(a) Simplify $\frac{2}{x+1}+\frac{2}{x^2-1}$ **1**

(b) Find rational numbers *a* and *b* such that $\left(5-\sqrt{3}\right)^2 = a+b\sqrt{3}$ **2**

(c) Solve the following pair of simultaneous equations:

$x+y-3=0$

$y-2x^2=0$ **2**

(d) Find the values of *k* for which the quadratic expression $kx^2+\left(k-1\right)x+k$ is positive definite. **2**

(e) Differentiate:

(i) $y = 6x - 8x^6 - 7$ **1**

(ii) $y = 4x^3 - \frac{6}{x^2}$ **2**

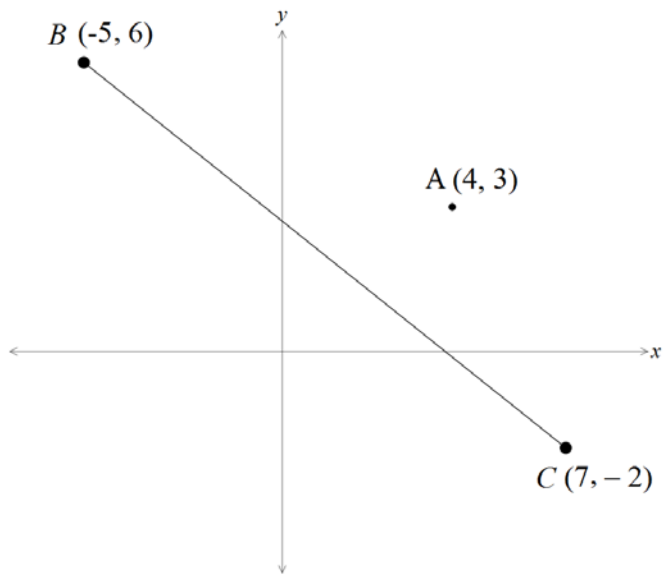
(iii) $y = \frac{2x}{\sqrt{x-1}}$ (leave your answer as one simplified fraction) **3**

(f) Sketch $y = \frac{4}{x-5} + 1$, clearly showing all asymptotes and the *y*-intercept **2**

END OF QUESTION 11

Question 12 **Start a new booklet** **15 Marks**

(a) In the diagram below, the coordinates of *A*, *B* and *C* are respectively $(4, 3)$, $(-5, 6)$ and $(7, -2)$



(i) Find the length of *BC* **1**

(ii) Show that the equation of the line *BC* is $2x+3y-8=0$ **2**

(iii) Find the perpendicular distance from *A* to the line *BC* **2**

(iv) Hence, find the area of ΔABC **1**

(b) Solve the following for $0° \le \theta \le 360°$

(i) $\dfrac{\cos\theta}{\sqrt{3}} = \dfrac{1}{2}$ **2**

(ii) $\sin\theta\cos\theta = \cos\theta$ **2**

(c) On a number plane, shade the region given by $x \le 0$ and $y \ge 0$ and $x^2 + y^2 > 5$ **2**

(d) Find the equation of the normal to the curve $y = 2x(3-x)^5$ when $x = 2$. **3**

END OF QUESTION 12

Question 13 **Start a new booklet** **15 Marks**

(a) Given that the roots of the quadratic equation $4x^2 - x + 6$ are α and β, find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(b) A plane flies from town *A* on a bearing of 272°. It travels for 450 km to town *B*. It leaves town *B* on a bearing of 286° and flies for 325 km to town *C*.

(i) Show this information in a diagram. **1**

(ii) Show that $\angle ABC$ = 166°, justifying your working with reasons. **2**

(iii) What is the distance from town *A* to town *C*, to the nearest kilometre? **2**

(iv) If the plane flies at an average speed of 210 km/h, how long will it take to go from Town C to Town A, correct to the nearest minute? **1**

(c) Solve $2\left(4^x\right) - 9\left(2^x\right) + 4 = 0$ **2**

(d) Differentiate $f\left(x\right) = 3x^2 + x - 1$ from first principles. **3**

END OF QUESTION 13

Question 14 **Start a new booklet** **15 Marks**

(a)

(i) Find $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$ **2**

(ii) Hence, sketch $y = \dfrac{x^2 + x - 6}{x - 2}$, showing any points of discontinuity. **1**

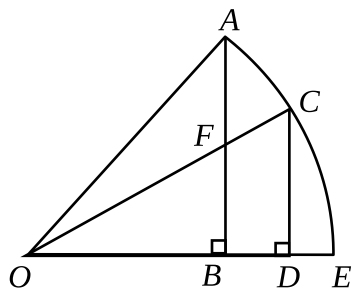
(b) Show that $\tan\theta + \cot\theta = \text{cosec}\,\theta \sec\theta$ **2**

(c) A concave up parabola has a vertex $(3, 1)$, and passes through $\left(5, \dfrac{3}{2}\right)$. Find:

(i) the focal length. **1**

(ii) the equation of the directrix **1**

(d) In the diagram, *ACE* is an arc of a circle with centre *O* and radius $2\sqrt{13}$ cm. $\angle OBA = \angle ODC = 90°$, $OB = CD = 4$ cm.



NOT TO SCALE

(i) Prove that ΔOAB is congruent to ΔOCD. **3**

(ii) Find the length of *BD*. **1**

(iii) Prove that ΔFOB is similar to ΔCOD **2**

(iv) Find the area of *BDCF*. **2**

END OF QUESTION 14

END OF EXAM

① $\frac{4}{4-2\sqrt{2}}$

$= \frac{~~4(4+2\sqrt{2})~~}{16-8}$

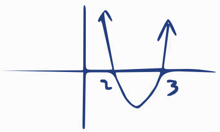
$= \frac{16+8\sqrt{2}}{8}$

$= 2+\sqrt{2}$

$\therefore$ (A)

② $x^2-5x+6<0$

$(x-3)(x-2)<0$



$2<x<3$ $\therefore$ (C)

③ $x^2-(1-\sqrt{2}+1+\sqrt{2})x+(1-\sqrt{2})(1+\sqrt{2})=0$

$x^2-(2)x+(1-2)=0$

$x^2-2x-1=0$

$\therefore$ (B)

④ $4x-3y+1=0$

$3y=4x+1$

$y=\frac{4x+1}{3}$

$\therefore m_{\underline{h}} = -\frac{3}{4}$

$\therefore$ (D)

⑤ $\frac{d}{dx}(1-2x)^5$

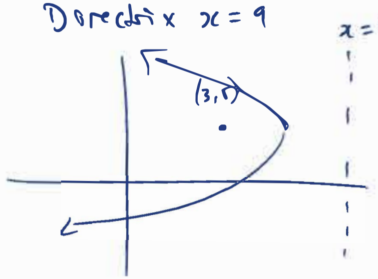
$=(5)(-2)(1-2x)^4$

$=-10(1-2x)^4$

$\therefore$ (D)

⑥ Focus: $(3,5)$

Directrix $x=9$



$\therefore$ focal ~~length~~ $=3$

$\therefore$ vertex $=(6,5)$

$\therefore (y-5)^2=-12(x-6)$

$y^2-10y+25=-12x+72$

$y^2-10y+12x-47=0$

$\therefore$ (B)

⑦ $2x^2=3y$ | $4a=3/2$

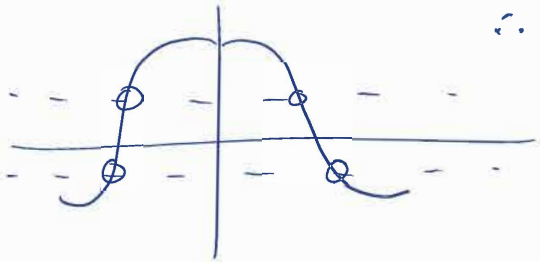
$x^2=3/2\,y$ | $a=3/8$

$\therefore$ Focus $(0, 3/8)$

$\therefore$ (D)

⑧ $2\cos^2\theta=1$

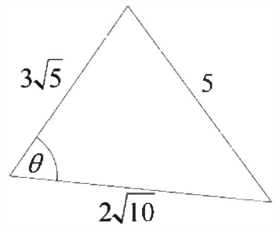
$\cos\theta=\pm\frac{1}{\sqrt{2}}$



$\therefore$ 4 ~~solutions~~

$\therefore$ (D)

⑨



$\cos = \dfrac{(3\sqrt{5})^2 + (2\sqrt{10})^2 - (\ )^2}{2(3\sqrt{5})(2\sqrt{10})}$

$\cos = \dfrac{45 + 4\ \ -25}{12\sqrt{5c}}$

$= \dfrac{6}{0\sqrt{2}}$

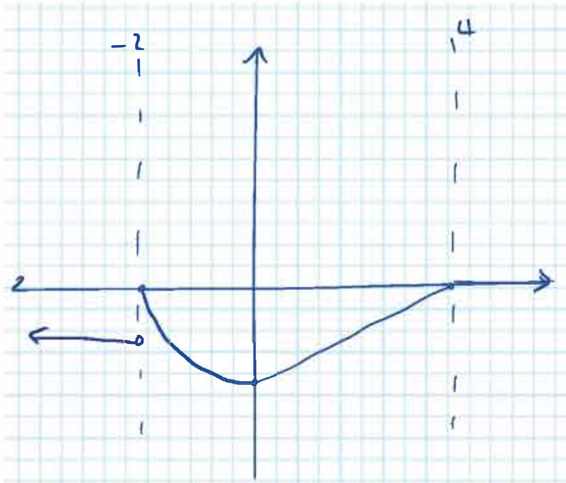
$\cos = \dfrac{1}{\sqrt{2}}$ $\therefore$ $= 45$

. (B)

⑩ $f(x) = \begin{cases} -1, & x < -2 \\ x^2 - 4, & -2 \le x < 0 \\ x - 4, & 0 \le x \le 4 \\ 0, & x > 4 \end{cases}$?

$\therefore\ -4 \le y <$

$\therefore$ (D)



(a) $\dfrac{2}{x+1} + \dfrac{2}{x^2-1}$

$= \dfrac{2x - 2 + 2}{x^2+1}$

$= \dfrac{2x}{x^2+1}$ — ①

(b) $(5 - \sqrt{3})^2 = a + b\sqrt{3}$

LHS $= 25 - 10\sqrt{3} + 3$ — ①

$= 28 - 10\sqrt{3}$

$\therefore\ a = 28,\ b = -10$ — ①

(c) $x + y - 3 = 0$ — ①

$y - 2x^2 = 0$

$y = 2x^2$ — ②

sub ② into ①

$x + 2x^2 - 3 = 0$

$2x^2 + x - 3 = 0$

$(2x + 3)(x - 1) = 0$

$\therefore\ x = -3/2,\ 1$

①

when $x = -3/2$

$y = 2\left(\frac{-3}{2}\right)^2$

$= 9/2$

when $x = 1$

$y = 2(1)^2$

$= 2$

$\therefore\ x = -3/2,\ y = 9/2$

or $x = 1,\ y = 2$ } ①

(d) $kx^2 + (k-1)x + k$

$a > 0$ $\Delta < 0$

$\therefore k > 0$

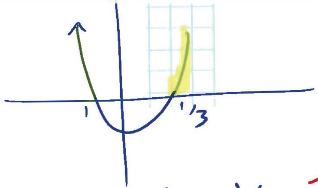
$(k-1)^2 - 4(k)(k) < 0$

$k^2 - 2k + 1 - 4k^2 < 0$

$-3k^2 - 2k + 1 < 0$

$3k^2 + 2k - 1 > 0$

$(3k-1)(k+1) > 0$



$k < -1$, $k > 1/3$ (1)

But! $k > 0$ since $a > 0$

$\therefore k > 1/3$ — (1)

(e)

(i) $y = 6x - 8x^6 - 7$

$y' = 6 - 48x^5$ — (1)

(e)

(ii) $y = 4x^3 - \frac{6}{x^2}$

$y = 4x^3 - 6x^{-2}$ (1)

$y' = 12x^2 + 12x^{-3}$ — (1)

$= 12x^2 + \frac{12}{x^3}$

(iii) $y = \frac{2x}{\sqrt{x-1}}$

$y = \frac{2x}{(x-1)^{1/2}}$

$y' = \frac{(2)(x-1)^{1/2} - (1/2)(x-1)^{-1/2}(2x)}{x-1}$ (1)

$= \frac{2\sqrt{x-1} - \frac{x}{\sqrt{x-1}}}{x-1}$ — (1)

$= \frac{2(x-1) - x}{\sqrt{x-1}} \div x - 1$

$= \frac{2x - 2 - x}{(x-1)(x-1)^{1/2}}$

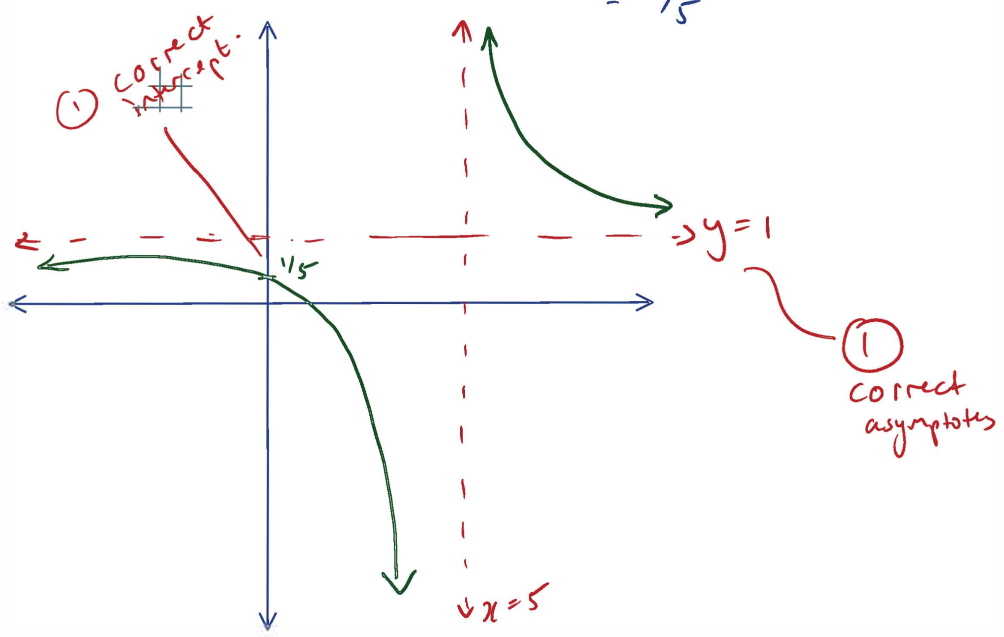
$= \frac{x-2}{(x-1)^{3/2}}$ — (1)

(f) $y = \frac{4}{x-5} + 1$

$x \neq 5$, $y \neq 1$

$x = 0$, $y = \frac{-4}{5} + 1$

$= 1/5$



(a) A(4,3) B(−5,6) C(7,−2)

(i) $BC = \sqrt{(7+5)^2 + (-2-6)^2}$

$= \sqrt{144 + 64}$

$= \sqrt{208}$ — ①

$= 4\sqrt{13}$ units

(ii) $M_{BC} = \frac{6-(-2)}{-5-7}$

$= \frac{8}{-12}$

$= -2/3$ — ①

$\therefore y - 6 = -\frac{2}{3}(x+5)$

$3y - 18 = -2x - 10$

$2x + 3y - 8 = 0$ } ①

(iii) $d = \frac{|2(4) + 3(3) - 8|}{\sqrt{2^2 + 3^2}}$ — ①

$= \frac{|9|}{\sqrt{13}}$

$= \frac{9}{\sqrt{13}}$ units. — ①

(iv) $A_{\Delta ABC} = \frac{1}{2} \times (4\sqrt{13}) \times \left(\frac{9}{\sqrt{13}}\right)$

$= 18$ units^2 — ①

(b) (i) $\frac{\cos\theta}{\sqrt{3}} = \frac{1}{2}$ $\quad 0° \le \theta \le 360°$

$\cos\theta = \sqrt{3}/2$

(1)

$\therefore \theta = 30°, 330°$ (1)

(ii) $\sin\theta\cos\theta - \cos\theta = 0$

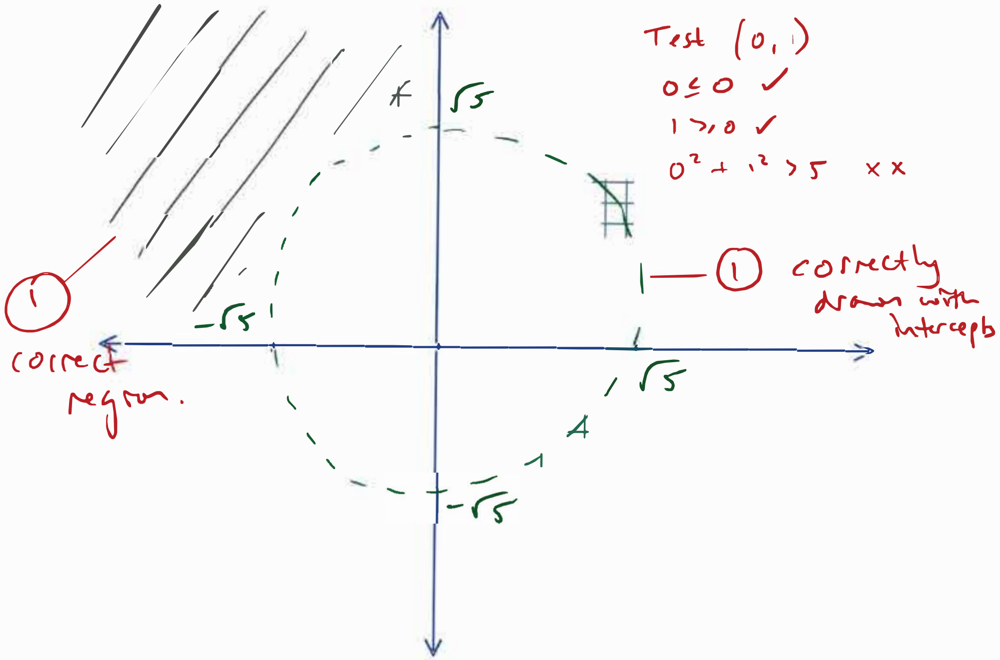
$\cos\theta(\sin\theta - 1) = 0$

$\cos\theta = 0$ $\quad \sin\theta = 1$

$\theta = 90°, 270°$ (1) $\quad \theta = 90°$

$\therefore \theta = 90°, 270°$ — (1)

(c) $x \le 0,\ y \ge 0,\ x^2 + y^2 > 5$



(d) $y = 2x(3-x)^5$ when $x = 2$

$\frac{dy}{dx} = (2)(3-x)^5 + (5)(-1)(3-x)^4(2x)$ — (1)

$= 2(3-x)^5 - 10x(3-x)^4$

when $x = 2$

$M_T = 2(1)^5 - 10(2)(1)^4$

$= 2 - 20$

$= -18$

$\therefore M_N = \frac{1}{18}$ — (1)

when $x = 2$, $y = 2(2)(3-2)^5$ $\quad \therefore (2, 4)$

$= 4$

$\therefore y - 4 = \frac{1}{18}(x - 2)$

$18y - 72 = x - 2$

$x - 18y + 70 = 0$ (1)

(a) $4x^2 - x + 6$

(i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{1}{4}$

(ii) $\alpha\beta = \frac{c}{a}$

$= 3/2$

(iii) $\alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$ — ①

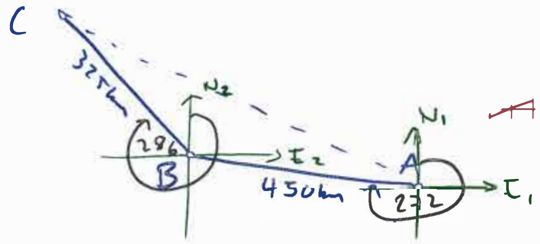
$= \left(\frac{1}{4}\right)^2 - 2\left(6/4\right)$

$= \frac{1}{16} - 3$

$= -\frac{47}{16}$ — ①

(b)

(i)



① correct diagram.

(ii) $\angle CBN_2 = 74°$ (angles in a revolution $= 360°$)

$\angle ABE_2 = 2°$ (alternate angles on parallel lines are $=$)

$\therefore \angle ABC = 74° + 2° + 90°$

$= 166°$ —— ① correct show ② sufficient justification.

(iii) $AC^2 = (325)^2 + (450)^2 - 2(325)(450)\cos 166$ — ①

$AC = 769$ km (nearest km) — ①

(iv) Time $= \frac{769 \text{ km}}{210 \text{ km/hr}}$

$=$ 3hrs 40 minutes (nearest min) ①

(c) $2(4^x) - 9(2^x) + 4 = 0$

$2(2^{2x}) - 9(2^x) + 4 = 0$

let $u = 2^x$

$\therefore 2u^2 - 9u + 4 = 0$ — (1)

$(2u - 1)(u - 4) = 0$

$u = 1/2 \qquad u = 4$

$\therefore 2^x = \frac{1}{2} \qquad 2^x = 4$

$\therefore x = -1, 2$ — (1)

(d) $f(x) = 3x^2 + x - 1$

$f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 + (x+h) - 1 - (3x^2 + x - 1)}{h}$ — (1)

$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + x + h - 1 - 3x^2 - x + 1}{h}$

$= \lim_{h \to 0} \frac{6xh + 3h^2 + h}{h}$ — (1)

$= \lim_{h \to 0} 6x + 3h + 1$

$= 6x + 1$ — (1)

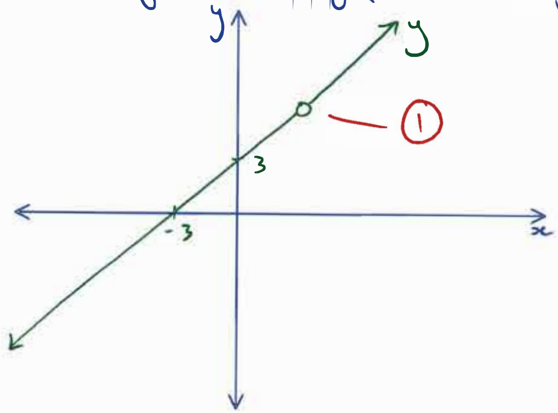
(a) (i) $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

$= \lim_{x \to 2} \frac{(x-2)(x+3)}{x-2}$ — (1)

$= \lim_{x \to 2} x + 3$

$= 5$ — (1)

(ii) Point of discontinuity @ $(2, 5)$ on $y = \frac{x^2 + x - 6}{x - 2}$



(b) $\tan\theta + \cot\theta = \operatorname{cosec}\theta \sec\theta$

LHS $= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta}$ — (1)

$= \frac{1}{\sin\theta\cos\theta}$

$= \operatorname{cosec}\theta \sec\theta$ — (1)

(c) Vertex (3, 1)   Point: (5, 3/2)

(i) $(x-3)^2 = 4a(y-1)$

$\therefore (5-3)^2 = 4a(3/2 - 1)$

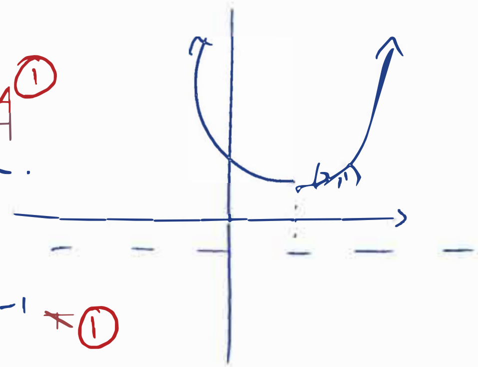
$4 = 4a(\frac{1}{2})$

$4 = 2a$

$a = 2$

$\therefore$ focal length = 2.

A (1)

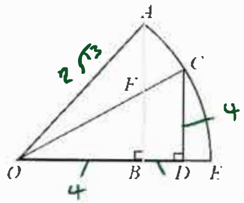


(ii) $(x-3)^2 = 8(y-1)$

$\therefore$ Directrix @ $y = -1$ (1)

(d)

In the diagram, ACE is an arc of a circle with centre O and radius $2\sqrt{13}$ cm.
$\angle OBA = \angle ODC = 90°$, $OB = CD = 4$ cm.

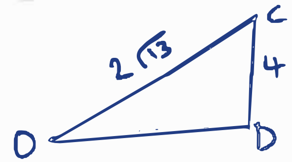


(i) In $\triangle OAB$ and $\triangle OCD$ → (1) correct setting

- $OB = CD$ (given) } (1)
- $\angle OBA = \angle ODC = 90°$ (given)
- $AO = OC$ (equal radii) — (1)

$\therefore \triangle OAB \equiv \triangle OCD$ (RHS)

(ii) In $\triangle OCD$



$OD^2 = (2\sqrt{13})^2 - (4)^2$

$OD = 6$ cm

$BD = OD - OB$

$= 2$ cm (1)

(iii) In $\triangle FOB$ and $\triangle COD$

- $\angle COD$ is common } (1)
- $\angle FBO = \angle CDO$ (given)

$\therefore \triangle FOB \,|||\, \triangle COD$ (equiangular) (1)

(iv) $\frac{FB}{CD} = \frac{OB}{OD}$ (matching sides of similar triangles are in ratio) — (1)

$FB = \frac{(4)(4)}{6}$

$= 8/3$

$\therefore A_{BDCF} = \frac{(2)}{2}(4 + 8/3)$

$= \frac{20}{3}$ cm² — (1)