SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS 2 UNIT

YEAR 11 PRELIMINARY COURSE

MAY 2014

NAME ______________________________ TEACHER ______________________________

Time allowed 75 minutes

Instructions * Begin each question on a new page.

* Marks shown are a guide and may be varied.

* Show necessary working.

* Full marks may not be awarded if your working is poorly set out or illegible.

* Leave all answers in simplest form.

* Use a ruler for all straight lines.

Question 1 (7 marks)

a) Evaluate $\sqrt{\frac{\pi}{1{\cdot}6^2}}$ correct to two significant figures. 1

b) Simplify i) $\sqrt{8}+\sqrt{18}$ 1

ii) $(3\sqrt{2}-4)^2$ 1

c) Evaluate $|-4| - |8-20|$ 1

d) Find the values of p and q such that $\frac{\sqrt{5}}{\sqrt{5}-2} = p + q\sqrt{5}$ 2

e) Simplify $2^x \div 2^{x-3}$ 1

Question 2 (7 marks) Start a new page.

a) Subtract $2x^2-3x-1$ from $2x^2-x+5$ 1

b) Simplify $\frac{(3m^2x^3)^2 \times 2mx^2}{4m^8x}$ 2

c) Rewrite $m(m+2)^{-1}$ without a negative index. 1

d) Fully factorise: i) $2x^2-18$ 1

ii) $3x^2+11x-4$ 1

iii) $a^2-ab-a+b$ 1

Question 3 (7 marks) Start a new page.

a) Simplify $\frac{a}{a+\frac{1}{a}}$ 1

b) Solve: i) $7(m-4) = 2(m+11)$ 1

ii) $\frac{6-4y}{2} < \frac{y}{3} + 2$ 2

c) Solve each quadratic equation, leaving answers in simplest exact form:

i) $3x-4x^2=0$ 1

ii) $4x^2-6x-1=0$ 2

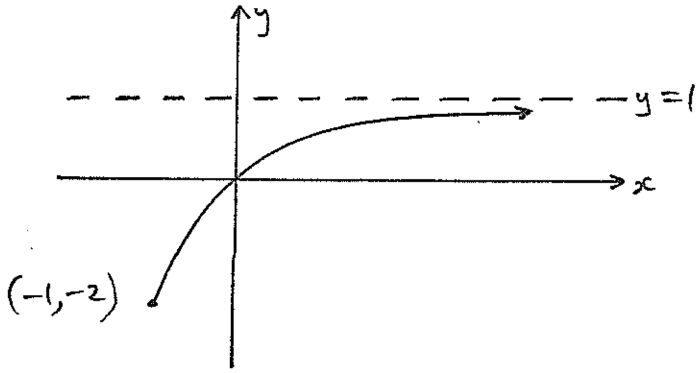
Question 4 (7 marks) Start a new page.

a) Solve $|2x+1| = 7$ 2

b) Given $H(x) = x^2 - 3x$, find and simplify: i) H(-1) 1

ii) $H(m+4)$ 2

c) Write the domain and range of the function graphed below: 2



Question 5 (7 marks) Start a new page.

a) Neatly sketch the parabola $y = x^2 - x - 6$. Clearly show intercepts on both axes and the coordinates of the vertex. 2

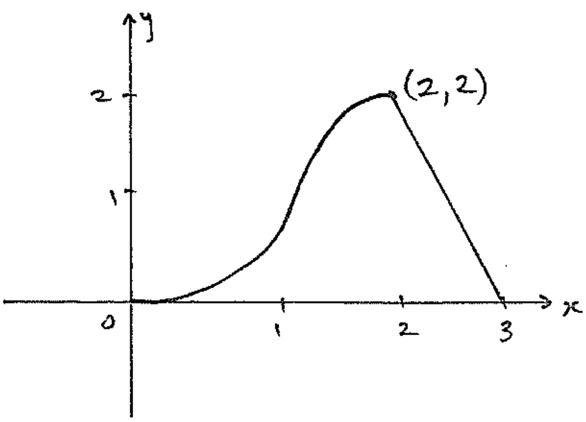
b) Sketch each curve below on separate number planes. Use a ruler and clearly label asymptotes or other key features. i) $x^2 + (y-2)^2 = 4$ 2

ii) $y = \frac{1}{x+2} + 1$ 2

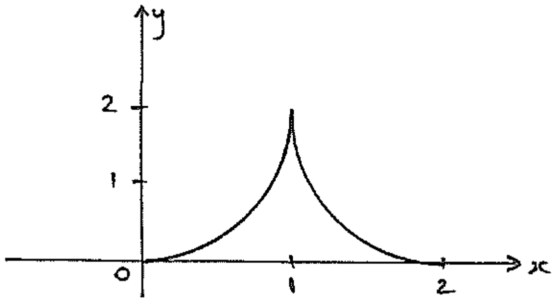
c) Given $\frac{1}{x\sqrt{x}} = x^a$, find the value of a. 1

Question 6 (7 marks) Start a new page.

a) Neatly copy the curve below into your answer booklet. Add a new section of the curve so that the total curve clearly represents an odd function. Label key points. 1



b) Given the curve $y = f(x)$ below:



i) Evaluate $f(2)$ 1

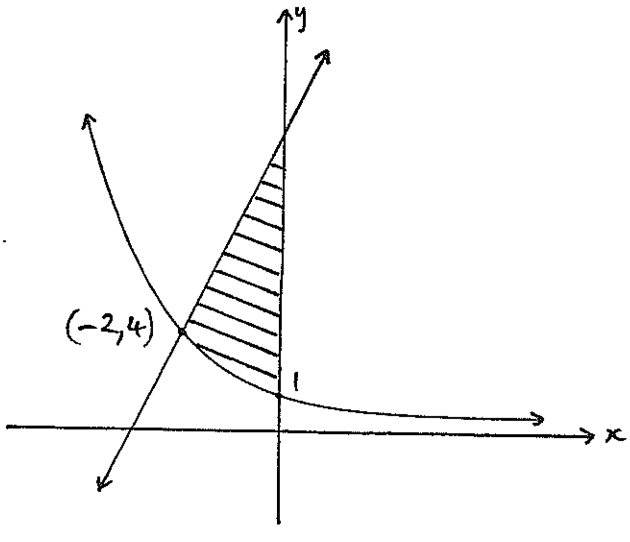
ii) Sketch the graph of $y = f(x) - 1$ 1

c) i) Sketch the graph of $y = |4 - x|$ 1

ii) Hence, or otherwise, solve $|4 - x| < 3$ 1

d) Simplify $\frac{x^3-1}{x^2-1}$ 2

Question 7 (7 marks) Start a new page.



The diagram, not to scale, shows the line $2x - y + 8 = 0$ and an exponential function of the form $y = a^{-x}$.

a) i) Write the equation of the exponential function shown. 1

ii) The shaded area represents the intersection of three (3) regions. Write the inequalities for these regions. 3

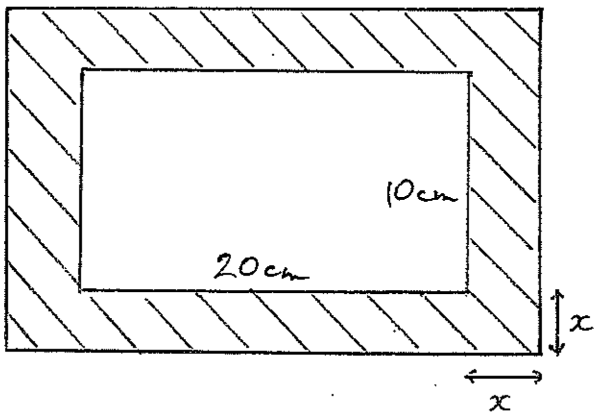
b) Solve simultaneously to find the points of intersection of the graphs of

$xy = 4$ and $2x - y - 2 = 0$ 3

Question 8 (7 marks) Start a new page.

a) Find the domain and range of the function $f(x) = 10 + \frac{1}{\sqrt{x-4}}$ 2

b)



A photograph measures 20 cm by 10 cm. It is surrounded by the shaded border of uniform width x cm. The total area of photo + border is 416 cm^2.

Write and solve a quadratic equation in x and find the dimensions of the border. 3

c) Find a value x such that $\sqrt{x + \sqrt{x + \sqrt{x + \cdots}}} = \frac{1+\sqrt{17}}{2}$ 2

(hint: you may want to start by squaring both sides)

END OF TEST

Solutions.

1. a) $1:1$

b) i) $2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$

ii) $18-24\sqrt{2}+16=34-24\sqrt{2}$

c) $4-12=-8$.

d) $\dfrac{\sqrt{5}}{\sqrt{5}-2}\times\dfrac{\sqrt{5}+2}{\sqrt{5}+2}=\dfrac{5+2\sqrt{5}}{5-4}$

$=5+2\sqrt{5}$

$\therefore p=5,\ q=2$

e) 2^3 or 8

2. a) $2x^2-x+5-2x^2+3x+1=2x+6$

b) $\dfrac{9m^4x^6\times 2mx^2}{4m^8x}=\dfrac{9x^7}{2m^3}$

c) $\dfrac{m}{m+2}$

d) i) $2(x+3)(x-3)$

ii) $(3x-1)(x+4)$

iii) $a(a-b)-(a-b)$

$=(a-b)(a-1)$

3. a) $\dfrac{a}{a^2+1/a}=a\times\dfrac{a}{a^2+1}$

$=\dfrac{a^2}{a^2+1}$

b) i) $7m-28=2m+22$

$5m=50$

$m=10$

ii) $\dfrac{6-4y}{2}\times 6^3<\dfrac{y}{3}\times 6^2+2\times 6$

$18-12y<2y+12$

$-14y<-6$

$y>3/7$

c) i) $x(3-4x)=0$

$x=0,\ 3/4$

ii) $x=\dfrac{6\pm\sqrt{36-4\times4\times-1}}{8}$

$=\dfrac{6\pm\sqrt{52}}{8}$

$=\dfrac{6\pm2\sqrt{13}}{8}$

$=\dfrac{3\pm\sqrt{13}}{4}$

④ a) $2x+1=7$ or $-2x-1=7$

$x=3$ $\quad\quad -2x=8$

$x=-4$

$\therefore x=3,-4$

b) i) $H(-1)=(-1)^2+3$

$=4$

ii) $H(m+4)=(m+4)^2-3(m+4)$

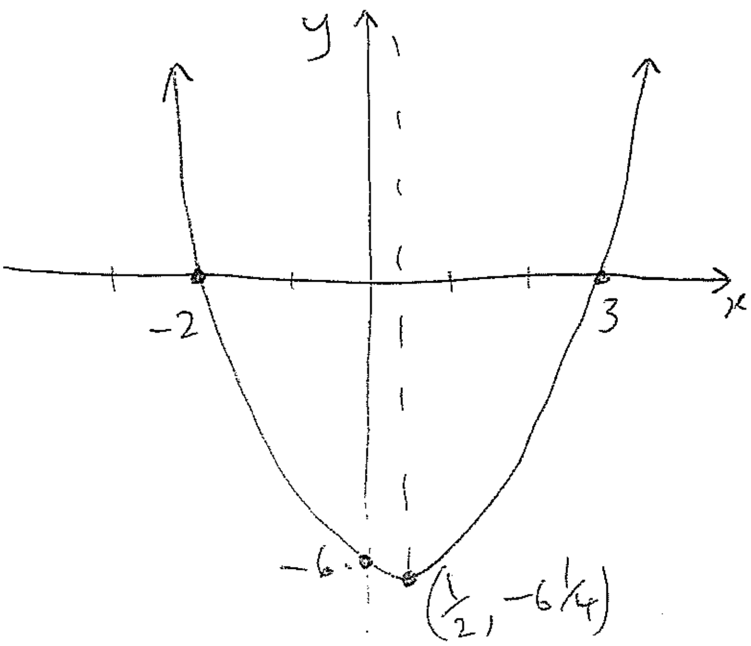
$=m^2+8m+16-3m-12$

$=m^2+5m+4$

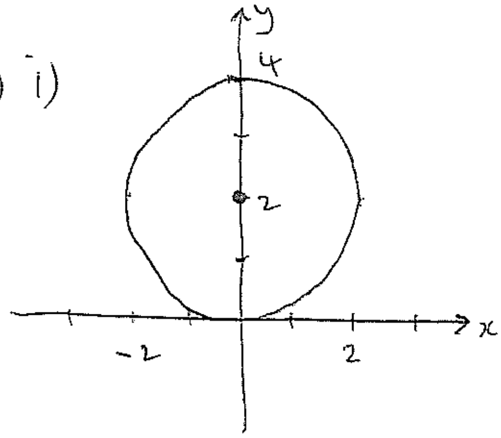
c) $D: x \geqslant -1$

$R: -2 \leq y < 1$

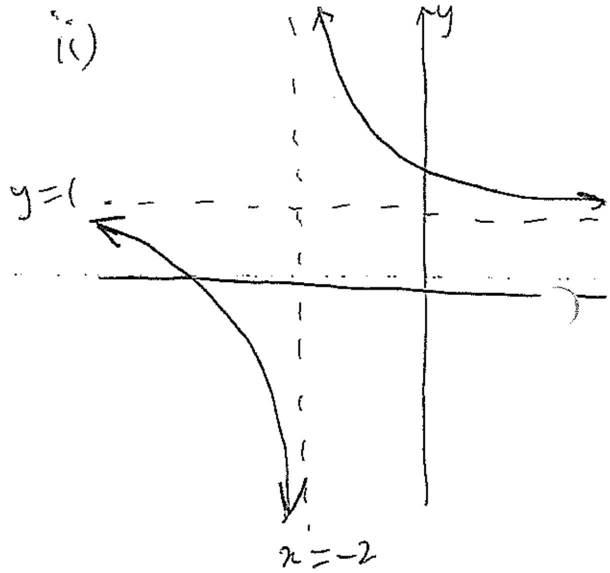
⑤ a) $y=(x-3)(x+2)$



b) i)



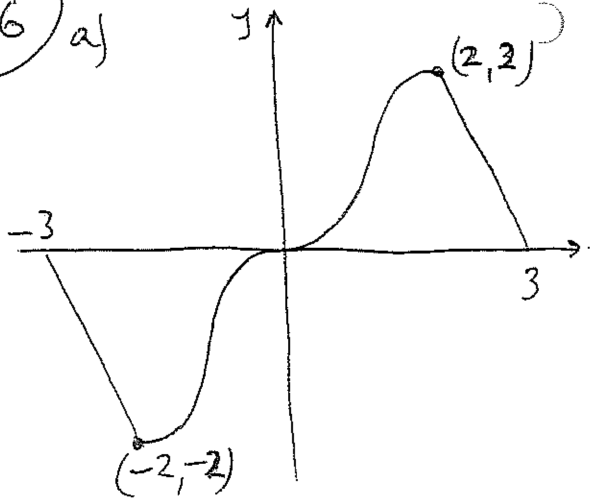
ii)



c) $\frac{1}{x^{3/2}} = x^{-3/2}$

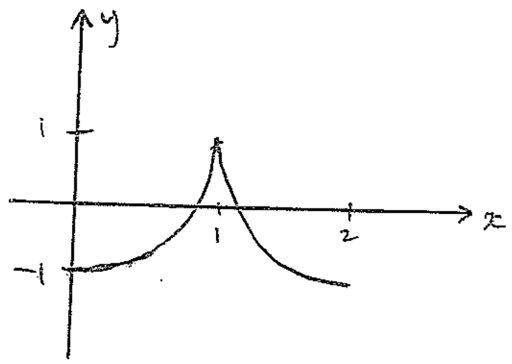
$\therefore a = -\frac{3}{2}$

⑥ a)

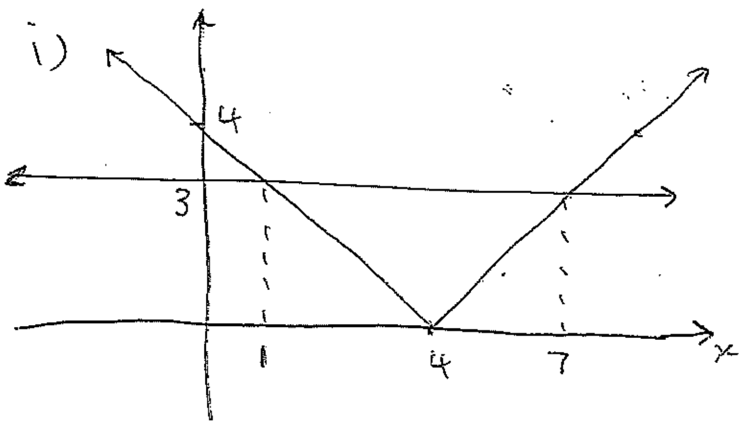


6 b) i) $f(2) = 0$

ii)



c) i)



ii) $1 < x < 7$

d) $\dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$

$= \dfrac{x^2+x+1}{x+1}$

7 a) i) $y = 2^{-x}$

ii) $x \leq 0$

$y \geq 2^{-x}$

$2x - y + 8 \geq 0$

b) $2x - y - 2 = 0$ — ①

$xy = 4$ — ②

From ① : $y = 2x - 2$

Sub ② : $x(2x-2) = 4$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2, -1$

$\therefore$ pts. of intersection are $(2,2)$ and $(-1,-4)$

8 a) $x - 4 > 0$

$D : x > 4$

$R : y > 10$

b) $(20+2x)(10+2x) = 416$

$200 + 40x + 20x + 4x^2 = 416$

$4x^2 + 60x - 216 = 0$

$x^2 + 15x - 54 = 0$

$(x-3)(x+18) = 0$

$\therefore x = 3, ~~-18~~$ no solution

$\therefore$ Border is 26 cm long
16 cm wide

⑧ c) (squaring) $x + \sqrt{x + \sqrt{x + \ldots}} = \left(\frac{1+\sqrt{17}}{2}\right)^2$

$$x + \frac{1+\sqrt{17}}{2} = \frac{1+2\sqrt{17}+17}{4}$$

$$\therefore x = \frac{18+2\sqrt{17}}{4} - \frac{2+2\sqrt{17}}{4}$$

$$= \frac{18+2\sqrt{17}-2-2\sqrt{17}}{4}$$

$$= 16/4$$

$$= 4.$$