Examination Number:

Set:

Shore

Year 11
Mathematics
Yearly Examination
September 2014

General Instructions

- Reading time − 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- Answer Questions 1 – 10 on the Multiple Choice answer sheet provided
- Start each of Questions 11 – 14 in a new writing booklet
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A"

Total marks − 70

Section I Pages 3 - 7

10 marks

- Attempt Questions 1 − 10
- Each question is worth 1 mark
- Allow about 10 minutes for this section

Section II Pages 8 − 12

60 marks

- Attempt Questions 11 - 14
- Each question is worth 15 marks
- Allow about 1 hour and 50 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I
10 marks
Attempt Questions 1 - 10
Allow about 10 minutes for this section

Use the multiple choice answer sheet.

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1 If $x = 3y^3 - 2$, what is the value of y when $x = 19$?

(A) $\sqrt[3]{\frac{17}{3}}$

(B) $\sqrt[3]{7}$

(C) $\sqrt[3]{21}$

(D) $\frac{\sqrt[3]{21}}{3}$

2 What is $\frac{\sqrt{3}}{5+2\sqrt{3}}$ as a fraction with a rational denominator?

(A) $\frac{6+5\sqrt{3}}{37}$

(B) $\frac{5\sqrt{3}-6}{37}$

(C) $\frac{6+5\sqrt{3}}{13}$

(D) $\frac{5\sqrt{3}-6}{13}$

3 What is the domain and range of the function $y = \sqrt{4 - x^2}$?

(A) Domain $-2 \le x \le 2$, Range $0 \le y \le 2$

(B) Domain $-2 \le x \le 2$, Range $-2 \le y \le 2$

(C) Domain $0 \le x \le 2$, Range $-4 \le y \le 4$

(D) Domain $0 \le x \le 2$, Range $0 \le y \le 4$

4 What is the gradient of the normal to the curve $y = \dfrac{3x^2 + 2}{x^2}$ at $x = 2$?

(A) -2

(B) $-\dfrac{1}{2}$

(C) $\dfrac{1}{2}$

(D) 2

5 What is the solution of $2^{x+1} = \dfrac{1}{64}$?

(A) $x = -7$

(B) $x = -5$

(C) $x = 5$

(D) $x = 7$

6 What is the derivative of $\left(4x^2 - 5\right)^3$?

(A) $3\left(4x^2 - 5\right)$

(B) $3\left(4x^2 - 5\right)^2$

(C) $24x\left(4x^2 - 5\right)^2$

(D) $12x^2\left(4x^2 - 5\right)^2$

7 What is the simplified expression for $\dfrac{x^3 - 1}{x^2 - 1} \times \dfrac{x^2 - 4x - 5}{4x^2 + 4x + 4}$?

(A) $\dfrac{x - 5}{4}$

(B) $\dfrac{x - 1}{4}$

(C) $\dfrac{x + 1}{4}$

(D) $\dfrac{x^2 + x + 1}{4}$

8 What is the solution of the inequality $\left|3 - 2x\right| < 5$?

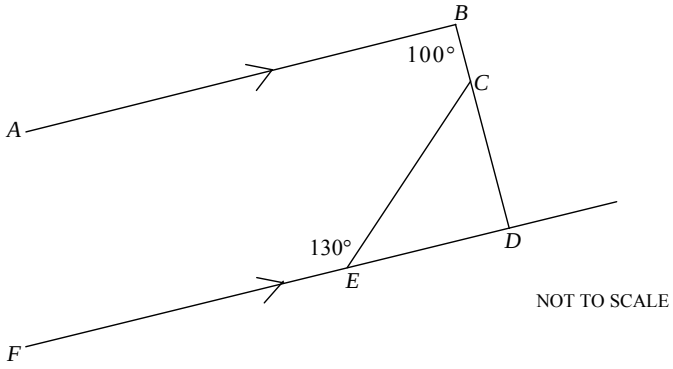
(A) $x < -4$ or $x > 1$

(B) $-4 < x < 1$

(C) $-1 < x < 4$

(D) $x < -1$ or $x > 4$

9 In the diagram below, AB is parallel to FD, $\angle ABC = 100°$ and $\angle CEF = 130°$.



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What is the value of $\angle BCE$?

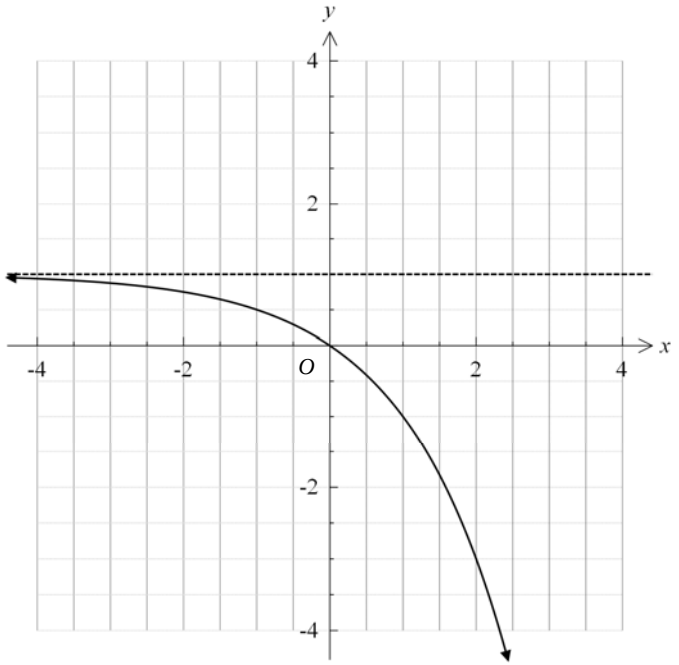
(A) 100°

(B) 110°

(C) 120°

(D) 130°

10 Which of the following equations represents this graph?



(A) $y = 2^x - 1$

(B) $y = 2^{-x} - 1$

(C) $y = 1 - 2^x$

(D) $y = 1 - 2^{-x}$

End of Section I

Section II

Total Marks 60
Attempt Questions 11 - 14
Allow about 1 hour and 50 minutes for this section.

Answer all questions, starting each question in a **new answer booklet** with your exam number clearly visible. Extra writing booklets are available.

In questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) A retailer marked up the wholesale price of a jacket by 37% before selling it for \$347.98. Calculate the wholesale price of the jacket to the nearest cent. **2**

(b) Solve $\frac{x-1}{3}-1=\frac{x+2}{2}$. **2**

(c) What angle does the tangent to the curve $y=x^3+x^2$ at $x=-1$ make with the positive direction of the x - axis? **2**

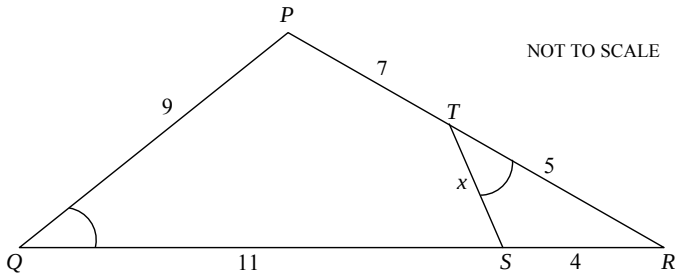
(d) Find the size of each interior angle of a regular octagon. **2**

(e) Evaluate $\lim\limits_{x\to\infty}\frac{3x^2-2x}{3x-5x^2}$. **2**

(f) Factorise fully x^4-16. **2**

Question 11 continues on the following page

(g)



In the diagram, $\angle PQR=\angle STR$.

(i) Prove that ΔPQR is similar to ΔSTR. **2**

(ii) Hence find the value of x. **1**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Differentiate $3x\sqrt{x}$. **2**

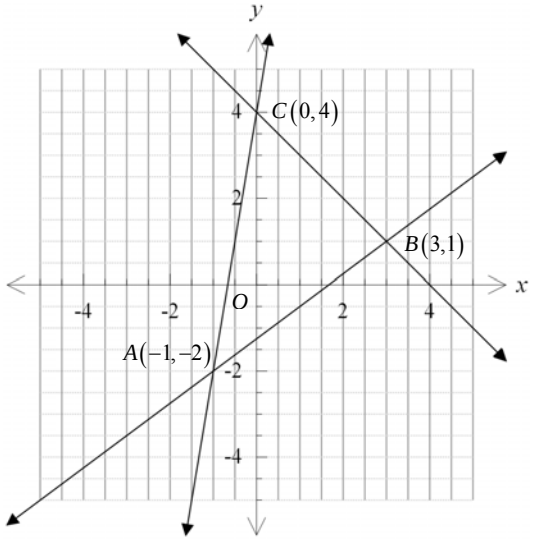
(b) If $y = x^3 - 4x^2 - 5$, for what values of x does $\frac{dy}{dx} = -4$? **3**

(c) A triangle has sides $7\,\text{cm}$, $9\,\text{cm}$ and $10\,\text{cm}$.

(i) Find the largest angle of the triangle. Answer to the nearest minute. **2**

(ii) Hence, or otherwise, find the area of the triangle correct to two significant figures. **1**

(d) The diagram below shows the points $A(-1,-2)$, $B(3,1)$ and $C(0,4)$.



(i) Find the centre and radius of the circle which has AB as diameter. **2**

(ii) Show that the equation of the line through A and B is $3x - 4y - 5 = 0$. **2**

(iii) Find the perpendicular distance from C to the line AB. **2**

(iv) Hence find the area of the triangle ABC. **1**

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) If α and β are the roots of $2x^2 - 3x - 4 = 0$, find

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha - \beta)^2$ **2**

(b) Solve $2\sin 2\theta + \sqrt{3} = 0$ for $0° \le \theta \le 360°$. **3**

(c) Solve $|x - 3| = 3x + 1$. **3**

(d) Find the values of k for which $x^2 + kx + 4k = 0$ has real roots. **3**

(e) Prove that $\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta} = \sec^2\theta \ \text{cosec}^2\theta$. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Find the equation of the tangent to the curve $y=(3x+1)(x-2)$ which is parallel to the line $7x-y-5=0$. **3**

(b) Differentiate $f(x)=x^2-3x$ from first principles. **3**

(c) If $y=x(2x-1)^3$, find $\frac{dy}{dx}$ in fully factored form. **3**

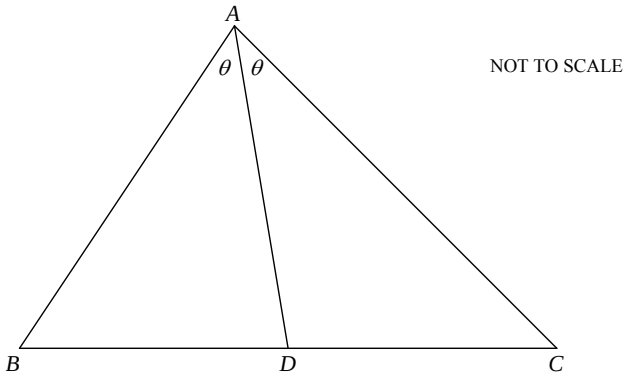
(d) Given the function $y=\frac{x^2}{x+1}$.

(i) Find $\frac{dy}{dx}$. **2**

(ii) Find the values of x for which the tangents to this curve are horizontal. **1**

(e) In triangle ABC, AD bisects $\angle BAC$. **3**

Use the sine rule to prove that $\frac{AB}{AC}=\frac{BD}{DC}$.



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End of Examination

YEAR 11 MATHEMATICS YEARLY EXAM SOLUTIONS

(1) $x = 3y^3 - 2$

$3y^3 = x + 2$

$y^3 = \frac{x+2}{3}$

$y = \sqrt[3]{\frac{x+2}{3}}$

$x = 19 \rightarrow y = \sqrt[3]{\frac{21}{3}} = \sqrt[3]{7}$

(B)

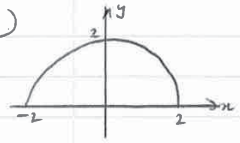
(2) $\frac{\sqrt{3}}{5+2\sqrt{3}} = \frac{\sqrt{3}}{5+2\sqrt{3}} \times \frac{5-2\sqrt{3}}{5-2\sqrt{3}}$

$= \frac{5\sqrt{3} - 2\times 3}{25 - 4\times 3}$

$= \frac{5\sqrt{3} - 6}{13}$

(D)

(3)



D: $-2 \le x \le 2$

R: $0 \le y \le 2$

(A)

(4) $y = 3 + \frac{2}{x^2} = 3 + 2x^{-2}$

$\frac{dy}{dx} = -4x^{-3} = -\frac{4}{x^3}$

When $x = 2$, $m_t = \frac{dy}{dx} = -\frac{4}{8} = -\frac{1}{2}$

$\therefore m_n = 2$

(D)

(5) $2^{x+1} = 2^{-6}$

$\therefore x + 1 = -6$

$x = -7$

(A)

(6) $y = (4x^2 - 5)^3$

$\therefore y' = 3(4x^2-5)^2 \times 8x = 24x(4x^2-5)^2$

(C)

(7) $\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)^2} \times \frac{(x-5)(x+5)}{4(x^2+x+1)}$

$= \frac{x-5}{4}$

(A)

(8) $|3 - 2x| < 5$

$-5 < 3 - 2x < 5$

$-8 < -2x < 2$

$4 > x > -1$ or $-1 < x < 4$

(C)

(9) $E\hat{D}C = 180° - 100°$ (cointerior $\angle$'s supplementary)

$= 80°$

$E\hat{C}D + 80° = 130°$ (Exterior Angle Theorem)

$E\hat{C}D = 50°$

$B\hat{C}E = 180° - 50°$ (straight angle)

$= 130°$

(D)

(10) $y = 1 - 2^x$ (C)

(11)

a) Let $x be wholesale price of jacket

$\therefore$ 137% of $x = 347.98$

$1.37x = 347.98$ (1)

$x = \frac{347.98}{1.37}$

$= 254$

$\therefore$ Wholesale price = $254 (1)

b) $\frac{x-1}{3} - 1 = \frac{x+2}{2}$

(x6) $2(x-1) - 6 = 3(x+2)$ (1)

$2x - 2 - 6 = 3x + 6$

$-14 = x$

$x = -14$ (1)

c) $y = x^3 + x^2$

$y' = 3x^2 + 2x$

When $x = -1$, $m_t = 3 - 2 = 1$ (1)

$\therefore \tan\theta = 1$

$\therefore \theta = 45°$ (1)

d) $S = (n-2) \times 180°$ $n = 8$

$= 6 \times 180°$

$= 1080°$ (1)

$\therefore$ each int $\angle = \frac{1080°}{8} = 135°$ (1)

e) $\lim_{x\to\infty} \frac{3x^2 - 2x}{3x - 5x^2} = \lim_{x\to\infty} \frac{3 - \frac{2}{x}}{\frac{3}{x} - 5}$ (1)

$= -\frac{3}{5}$ (1)

f) $x^4 - 16 = (x^2-4)(x^2+4)$ (1)

$= (x-2)(x+2)(x^2+4)$ (1)

g) i) In Δ's PQR, STR

$P\hat{Q}R = S\hat{R}T$ (common $\angle$)

$\frac{QR}{RT} = \frac{15}{5} = 3$

$\frac{PR}{SR} = \frac{12}{4} = 3$

(1)

$\therefore \Delta PQR \parallel\parallel \Delta STR$ (one $\angle$ equal, sides about $\angle$ proportional) (1)

OR

In Δ's PQR, STR

$P\hat{Q}R = S\hat{T}R$ (given)

$Q\hat{R}P = S\hat{R}T$ (same $\angle$) (1)

$\therefore \Delta PQR \parallel\parallel \Delta STR$ (equiangular) (1)

ii) $\frac{x}{9} = \frac{4}{12}$ (corresp sides proportional)

$\therefore x = \frac{36}{12} = 3$ (1)

(12) a) $y = 3x\sqrt{x} = 3x^1 \cdot x^{1/2} = 3x^{3/2}$ (1)

$\frac{dy}{dx} = \frac{9}{2}x^{1/2} = \frac{9}{2}\sqrt{x}$ or $\frac{9\sqrt{x}}{2}$ (1)

b) $y = x^3 - 4x^2 - 5$

$\frac{dy}{dx} = 3x^2 - 8x$ (1)

$\frac{dy}{dx} = -4 \longrightarrow 3x^2 - 8x = -4$

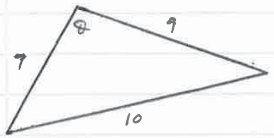
$3x^2 - 8x + 4 = 0$

$(3x-2)(x-2) = 0$ (1)

$\therefore 3x - 2 = 0$ or $x - 2 = 0$

$x = \frac{2}{3}, 2$ (1)

c) i) largest angle is opposite longest side (½)



$\cos\theta = \frac{7^2 + 9^2 - 10^2}{2 \cdot 7 \cdot 9}$

$= 0.238095238\ldots$ (1)

$\therefore \theta = 76°14'$ (½)

ii) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \cdot 7 \cdot 9 \cdot \sin 77°31'$

$= 30.755\ldots$

$= 31\ cm^2$ (2 s.f.)

(d) i) $AB = \sqrt{(3--1)^2 + (1--2)^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16+9}$

$= \sqrt{25} = 5u$ (1)

MP of AB: $x = \frac{x_1 + x_2}{2}$ $y = \frac{y_1 + y_2}{2}$

$= \frac{-1+3}{2}$ $= \frac{-2+1}{2}$

$= 1$ $= -\frac{1}{2}$ (1)

$\therefore$ Centre is $(1, -\frac{1}{2})$, radius $= \frac{5}{2}u$

ii) $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y+2}{x+1} = \frac{3}{4}$ (1)

$4y + 8 = 3x + 3$

$\therefore 3x - 4y - 5 = 0$ (1)

iii) $p = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

$= \frac{|3(0) - 4(4) - 5|}{\sqrt{9+16}}$ (1)

$= \frac{|-21|}{5}$

$= \frac{21}{5}u$ (1)

iv) Area $= \frac{1}{2}th$

$= \frac{1}{2} \times 5 \times \frac{21}{5}$

$= \frac{21}{2}$

$= 10\frac{1}{2}u^2$ (1)

(13) (a) $2x^2 - 3x - 4 = 0$

i) $\alpha + \beta = -\frac{b}{a} = \frac{3}{2}$ (1)

ii) $\alpha\beta = \frac{c}{a} = \frac{-4}{2} = -2$ (1)

iii) $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$= (\alpha^2 + \beta^2) - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$

$= (\alpha + \beta)^2 - 4\alpha\beta$ (1)

$= \frac{9}{4} - 4 \cdot -2$

$= 10\frac{1}{4}$ (1)

(b) $2\sin 2\theta = -\sqrt{3}$

$\sin 2\theta = -\frac{\sqrt{3}}{2}$ (½)

B.A $= 60°$ (½)

$\sin 2\theta < 0$ in 3rd, 4th quad.

$\therefore 2\theta = (180° + 60°), (360° - 60°)$.

$= (540° + 60°), (720° - 60°)$

$= 240°, 300°, 600°, 660°$ (1)

$\therefore \theta = 120°, 150°, 300°, 330°$ (1)

(c) $|x - 3| = 3x + 1$

$x - 3 = 3x + 1$ $x - 3 = -(3x+1)$

$-4 = 2x$ $x - 3 = -3x - 1$

$x = -2$ $4x = 2$

$x = \frac{1}{2}$ (1,1)

Test: $x = -2$ LHS $= |-5| = 5$

RHS $= -6 + 1 = -5$

$=$ not a soln

$x = \frac{1}{2}$ LHS $= |-2\frac{1}{2}| = 2\frac{1}{2}$

RHS $= 1\frac{1}{2} + 1 = 2\frac{1}{2}$

$\therefore x = \frac{1}{2}$ is a soln (1)

$\therefore x = \frac{1}{2}$

(d) $x^2 + kx + 4k = 0$

$\Delta = b^2 - 4ac$

$= k^2 - 4 \cdot 4k$

$= k^2 - 16k$ (1)

Real roots $\longrightarrow \Delta \geq 0$ (1)

$k^2 - 16k \geq 0$

$k(k - 16) \geq 0$

$\therefore k \leq 0$ or $k \geq 16$ (1)

(e) LHS $= \frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta \cos^2\theta}$

$= \frac{1}{\sin^2\theta \cos^2\theta}$ (1)

$= \frac{1}{\sin^2\theta} \cdot \frac{1}{\cos^2\theta}$

$= \csc^2\theta \sec^2\theta$

$=$ RHS (1)

(10) a) $y = (3x+1)(x-2) = 3x^2 - 5x - 2$

$\frac{dy}{dx} = 6x - 5$ (½)

Gradient of given line $= \frac{-7}{-1} = 7$ (½)

We require $\frac{dy}{dx} = 7$

$6x - 5 = 7$

$6x = 12$

$\therefore x = 2$

When $x = 2$, $y = 7 \times 0 = 0$ (1)

Eqn of tgt: $(2, 0)$, $m = 7$

$y - y_1 = m(x - x_1)$

$y - 0 = 7(x - 2)$

$y = 7x - 14$ (1)

(b) $f(x) = x^2 - 3x$

$f(x+h) = (x+h)^2 - 3(x+h)$

$= x^2 + 2xh + h^2 - 3x - 3h$

$f(x+h) - f(x) = x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)$

$= 2xh + h^2 - 3h$ (1)

$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 - 3h}{h}$

$= 2x + h - 3$ (1)

$f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$

$= \lim_{h \to 0} [2x + h - 3]$

$= 2x - 3$ (1)

(c) $y = x(2x-1)^3$

$y' = x \cdot 3(2x-1)^2 \cdot 2 + (2x-1)^3 \cdot 1$ (1)

$= 6x(2x-1)^2 + (2x-1)^3$

$= (2x-1)^2 [6x + 2x - 1]$ (1)

$= (8x-1)(2x-1)^2$ (1)

(d) $y = \frac{x^2}{x+1}$

(i) $\frac{dy}{dx} = \frac{(x+1) \cdot 2x - x^2 \cdot 1}{(x+1)^2}$

$= \frac{2x^2 + 2x - x^2}{(x+1)^2}$ (1)

$= \frac{x^2 + 2x}{(x+1)^2}$ (1)

(ii) We require $\frac{dy}{dx} = 0$

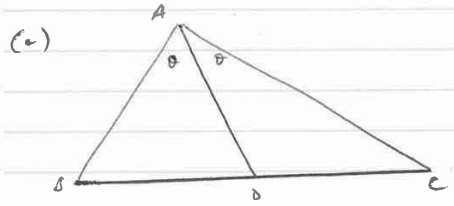
$\therefore \frac{x^2 + 2x}{(x+1)^2} = 0$

$\therefore x^2 + 2x = 0$

$x(x+2) = 0$ (1)

$\therefore x = -2, 0$

(e)



Let $A\hat{D}B = \alpha$ $\therefore A\hat{D}C = 180° - \alpha$.

In ΔABD, $\frac{BD}{AB} = \frac{\sin\theta}{\sin\alpha}$

In ΔACD $\frac{DC}{AC} = \frac{\sin\theta}{\sin(180° - \alpha)} = \frac{\sin\theta}{\sin\alpha}$

$\therefore \frac{BD}{AB} = \frac{DC}{AC}$

$\therefore BD \cdot AC = DC \cdot AB$

$\frac{BD}{BC} = \frac{AB}{AC}$

(1) for sine rule once
(1) for $\sin(180° - \theta) = \sin\alpha$
(1) correct ratios

OR In ΔABD, $\frac{\sin\theta}{BD} = \frac{\sin B}{AD}$

$\therefore \sin B = \frac{AD \sin\theta}{BD}$ ----(1)

In ΔADC $\frac{\sin\theta}{DC} = \frac{\sin C}{AD}$

$\sin C = \frac{AD \sin\theta}{DC}$ ---(2)

In ΔABC $\frac{\sin B}{AC} = \frac{\sin C}{AB}$

$\therefore \frac{\sin C}{\sin B} = \frac{AB}{AC}$

From (1) and (2), $\frac{AB}{AC} = \frac{\frac{AD \sin\theta}{DC}}{\frac{AD \sin\theta}{BD}}$

$= \frac{BD}{DC}$

(1) for sine rule once
(1) for sine rule another two times
(1) for correct ratios