KAMBALA

Student Number: _____

Task 4
September 2014

Preliminary HSC Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours and 30 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in Questions 9 - 13

Total marks – 83

Section I
8 marks

- Attempt Questions 1 - 8
- Allow about 15 minutes for this section

Section II
75 marks

- Attempt Questions 9 - 13
- Allow about 2 hours and 15 minutes for this section

Section I

8 Marks
Attempt Questions 1 – 8
Allow about 15 minutes for this section

Use the answer sheet for Questions 1 – 8.

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1. What is $\frac{2}{3-\sqrt{2}}$ as a fraction with a rational denominator?

   (A) $\frac{6-2\sqrt{2}}{7}$

   (B) $\frac{6+2\sqrt{2}}{7}$

   (C) $6-2\sqrt{2}$

   (D) $6+2\sqrt{2}$

2. What is the value of $f(-1)$ if $f(x)=x^2-4x$?

   (A) $f(-1)=-3$

   (B) $f(-1)=-5$

   (C) $f(-1)=3$

   (D) $f(-1)=5$

3. Select the axis of symmetry of the parabola $y=5x^2-4x-3$.

   (A) $x=\frac{4}{5}$

   (B) $x=\frac{2}{5}$

   (C) $x=-\frac{3}{5}$

   (D) $x=-\frac{4}{5}$

4. Which of the following is true for the equation $3x^2 - 5x - 2 = 0$?

(A) no real roots

(B) one real root

(C) two rational distinct roots

(D) two irrational distinct roots.

5. Find the value of b if $\sin 2b = \cos(b + 30^\circ)$

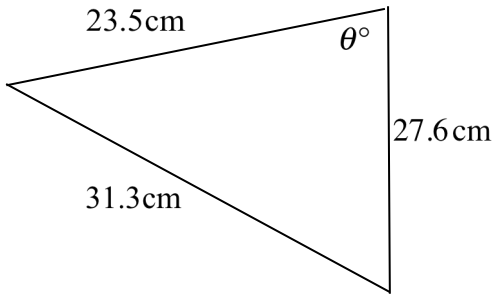
(A) 20°

(B) 30°

(C) 40°

(D) 50°

6. Which of the following is **NOT** a correct expression involving θ in ΔABC? **1**



Not to Scale

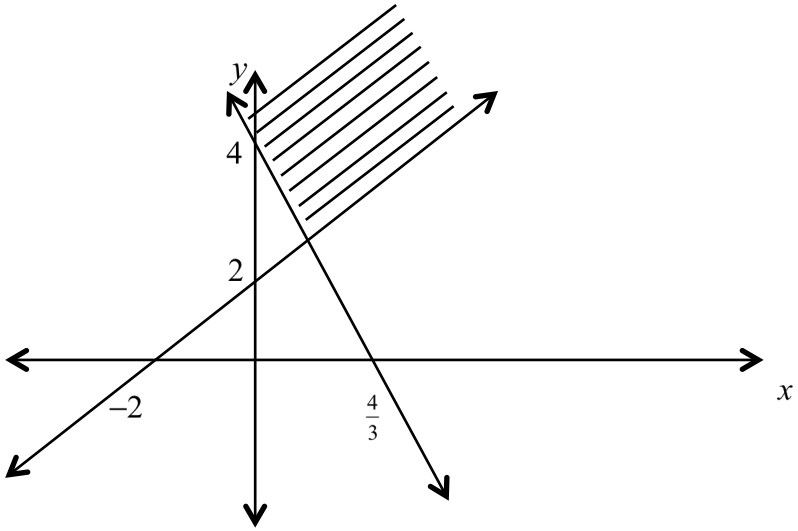
(A) $31.3^2 = 27.6^2 + 23.5^2 - 2 \times 27.6 \times 23.5 \cos\theta$

(B) $\cos\theta = \dfrac{23.5^2 + 27.6^2 - 31.3^2}{2 \times 23.5 \times 27.6}$

(C) $\dfrac{31.3}{\sin\theta} = \dfrac{27.6}{\sin 58^\circ 26'}$

(D) $\dfrac{\sin\theta}{31.3} = \dfrac{\sin 58^\circ 26'}{23.5}$

7. Which pair of inequalities describes the shaded region?



Not to scale

(A) $\begin{aligned} y &\geq 4-3x \\ y &\geq x+2 \end{aligned}$

(B) $\begin{aligned} y &\geq 4-3x \\ y &\leq x+2 \end{aligned}$

(C) $\begin{aligned} y &\leq 4-3x \\ y &\geq x+2 \end{aligned}$

(D) $\begin{aligned} y &\leq 4-3x \\ y &\leq x+2 \end{aligned}$

8. If $\text{cosec}\,\theta = -\frac{5}{3}$ and $\cos\theta > 0$, find $\cot\theta$.

(A) $\cot\theta = \frac{3}{4}$

(B) $\cot\theta = \frac{4}{3}$

(C) $\cot\theta = -\frac{3}{4}$

(D) $\cot\theta = -\frac{4}{3}$

End of Section I

Section II

75 Marks
Attempt Questions 9 - 13
Allow about 2 hours 15 minutes for this section

Answer each question on the writing paper provided. Start each question on a new page.
In Questions 9 - 13, your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (15 marks)

(a) Evaluate $\sqrt{\dfrac{2.3^2+3.7^2}{7.5}}$ correct to 2 significant figures. **2**

(b) Factorise x^3-125. **1**

(c) Solve $|2x-3|=7$ for x. **2**

(d) Consider the function $f(x)=x^2+2x-3$.

(i) Find the x and y intercepts. **2**

(ii) Find the minimum value of the function. **2**

(iii) Sketch the function. **1**

(iv) For what values of x is the curve decreasing? **1**

(e) Sketch the locus of the point that moves so that it is always 2 units from the point $P(4,2)$. **2**

(f) If $f(x)=x^2+3x-10$ find:

(i) $f'(x)$. **1**

(ii) the value of x for which $f'(x)=0$. **1**

Question 10 (15 marks) **Start a new page.**

(a) Find the exact value of $\tan 300^\circ$. **2**

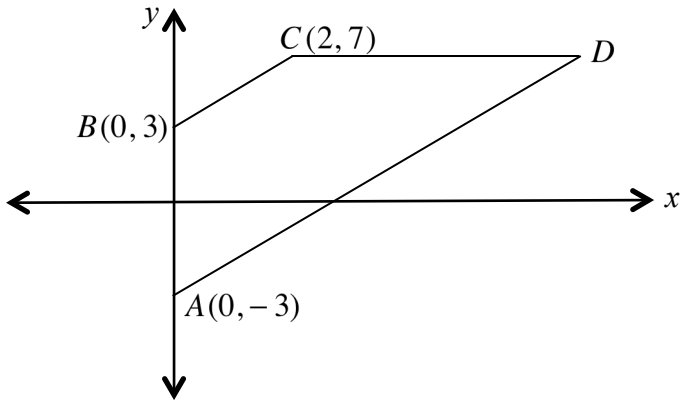
(b) In solving a quadratic equation, a student wrote down $x = \dfrac{4 \pm \sqrt{16+96}}{6}$. **2**

What was the original quadratic equation?

(c) (i) Solve the inequality $(x+10)(x+2) \geq 0$ for x. **2**

(ii) Hence state the domain of $\sqrt{x^2+12x+20}$. **1**

(d) In the diagram, $ABCD$ is a quadrilateral. The equation of the line AD is $2x - y - 3 = 0$.



Not to scale

(i) The line CD is parallel to the x-axis. Find the coordinates of D. **1**

(ii) Show that $ABCD$ is a trapezium by showing that BC is parallel to AD. **2**

(iii) Find the exact lengths of BC and AD. **2**

(iv) Show that the perpendicular distance from B to AD is $\dfrac{6}{\sqrt{5}}$ units. **2**

(v) Hence find the area of the trapezium $ABCD$. **1**

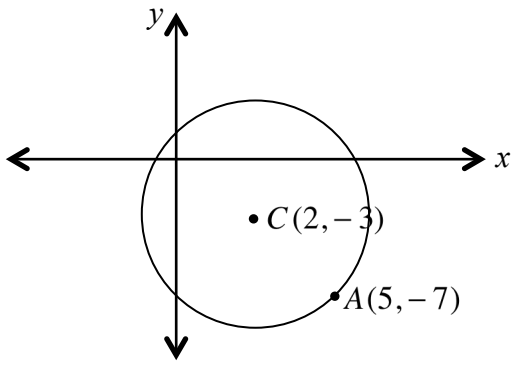
Note: $A = \dfrac{1}{2}(a+b)h$

Question 11 (15 marks) **Start a new page.**

(a) Simplify $\frac{m^2-4}{mn} \times \frac{2m}{2m-4}$. **2**

(b) Explain why $x^2+6x+11$ is positive definite. **2**

(c) A circle with centre $C(2,-3)$ has one end of a diameter at $A(5,-7)$. Find the coordinates of the other end of the diameter. **2**



Not to scale

(d) Let α and β be the roots of the equation $3x^2+5x-1=0$. Find:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha+1)(\beta+1)$ **2**

(e) Differentiate the following with respect to x:

(i) x^{-3} **1**

(ii) $(x-3)(2x+1)$ **2**

(iii) $\frac{5x^6-7x^2}{x}$ **2**

Question 12 (15 marks) **Start a new page.**

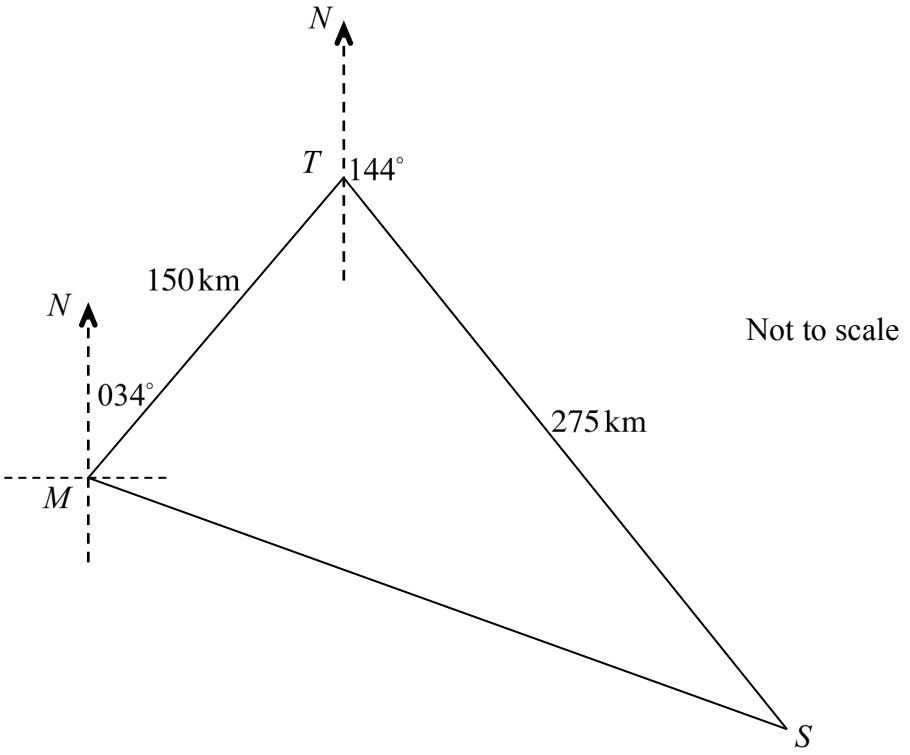
(a) A parabola has focus $S(2,-1)$ and the equation of the directrix is $x=-4$.

(i) Mark this information on a diagram and find the coordinates of the vertex. **2**

(ii) Write down the equation of the parabola. **1**

(b) Solve the equation $(x^2-2)^2-4(x^2-2)-21=0$ for x. **3**

(c) A ship sails from Melbourne, *M*, for 150 kilometres on a bearing of 034° to point *T*. It then sails on a bearing of 144° for 275 kilometres to point *S* as shown in the diagram below.



Copy the above diagram neatly onto your page.

(i) Show that $\angle MTS = 70^\circ$. **1**

(ii) How far, to the nearest kilometre, is the ship at point *S*, from Melbourne, *M*? **2**

(iii) What is the bearing of the ship, *S*, from Melbourne, *M*, to the nearest degree? **2**

Question 12 continued

(d) A function is given by $f(x) = x^2 - 1$.

(i) Find $f(2)$ and $f(2+h)$. **2**

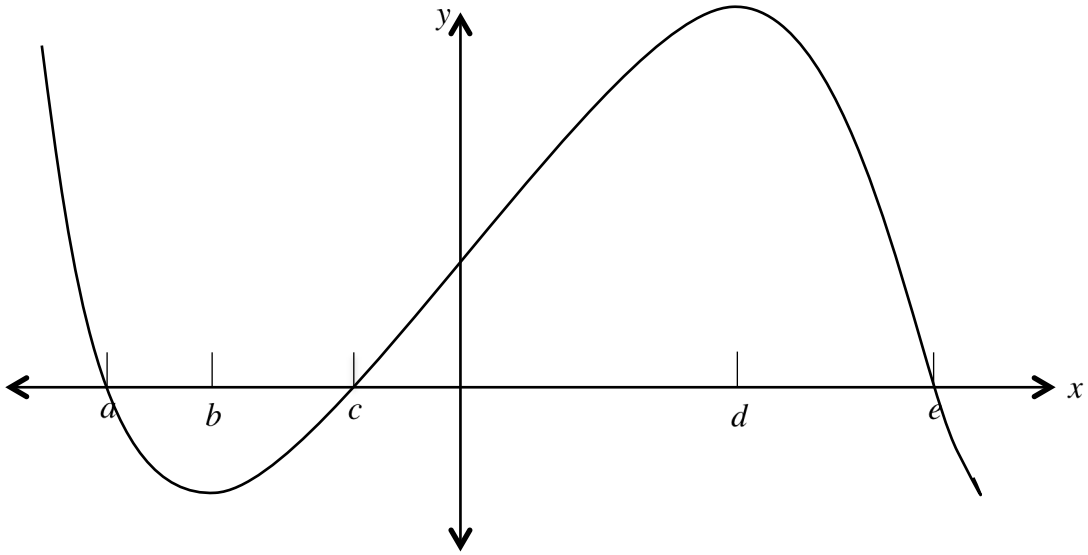
(ii) Show that $\dfrac{f(2+h)-f(2)}{h} = 4 + h$. **1**

(iii) Hence, or otherwise, find $f'(2)$. **1**

Question 13 (15 marks) **Start a new page.**

(a) If $2x^2 - 3x - 4 \equiv A(x+2)^2 + B(x+2) + C$ for all values of x, find the values of A, B and C. **3**

(b) The curve below represents the graph of function $y = f(x)$.



(i) Is the function odd, even or neither? **1**

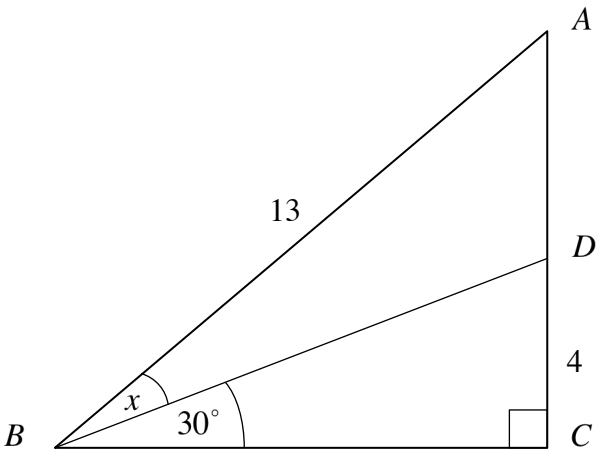
(ii) At what point(s) is $f(x) = 0$? **1**

(iii) Where is the gradient of the function equal to zero? **1**

(iv) Where is the gradient of the function positive? **1**

Question 13 continued

(c) The right-angled triangle ABC has hypotenuse $AB = 13$. The point D is on AC such that $DC = 4$, $\angle DBC = 30^\circ$ and $\angle ABD = x$.



Not to scale

(i) Find the length of BD. **1**

(ii) Using the sine rule, or otherwise, find the exact value of sin x. **3**

(d) (i) If the line $y = x + m$ cuts the circle $x^2 + y^2 = 4$, show that the x-coordinates of the points of intersection can be found by solving $2x^2 + 2mx + m^2 - 4 = 0$. **1**

(ii) For what value(s) of m will the line $y = x + m$ be a tangent to the circle? **3**

End of Section II

Kambala Preliminary HSC Mathematics
Assessment Task 4
September 2014
SOLUTIONS

1. $\frac{2}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$

$= \frac{6+2\sqrt{2}}{7}$ (B)

2. $f(x) = x^2 - 4x$

$f(-1) = (-1)^2 - 4(-1)$

$= 1 + 4$

$= 5$ (D)

3. $y = 5x^2 - 4x - 3$

$x = \frac{-b}{2a}$

$x = \frac{4}{2(5)}$

$x = \frac{2}{5}$ (B)

4. $3x^2 - 5x - 2 = 0$

$\Delta = (-5)^2 - 4(3)(-2)$

$= 25 + 24$

$= 49$

$\therefore \Delta > 0 \Rightarrow$ two distinct roots

Δ is a perfect square

$\Rightarrow$ rational roots (C)

5. $\sin 2b = \cos(b+30)$

$2b + b + 30 = 90°$

$3b + 30 = 90$

$3b = 60$

$b = 20°$ (A)

6. D (using Cosine Rule, angle btw 23.5 and 31.3 is 58°26')

7. $y \geq 4 - 3x$

test (0,0)

$0 \geq 4$

false

$\therefore$ region above $y = 4 - 3x$

$\therefore y \geq 4 - 3x$

$y \geq x + 2$

test (2,5)

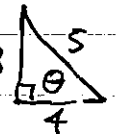
$5 \geq 2 + 2$

true

$\therefore y \geq 4 - 3x, y \geq x + 2$ (A)

8. $\operatorname{cosec}\theta = -\frac{5}{3}$, $\cos\theta > 0$

Quadrant 4



$\cot\theta = -\frac{4}{3}$ (D)

Question 9

a) $\sqrt{\frac{2 \cdot 3^2 + 3 \cdot 7^2}{7 \cdot 5}}$

$= \sqrt{\frac{5 \cdot 29 + 13 \cdot 69}{7 \cdot 5}}$

$= \sqrt{\frac{18 \cdot 98}{7 \cdot 5}}$

$= \sqrt{2 \cdot 5306}$

$\doteq 1 \cdot 5908$

$= 1 \cdot 6$ (to 2 sig figs)

b) $x^3 - 125$

$= (x-5)(x^2+5x+25)$

c) $|2x-3| = 7$

$2x-3=7$ or $3-2x=7$

$2x=10$ $\quad -2x=4$

$x=5$ $\quad x=-2$

$\therefore x = -2, 5$

d) $f(x) = x^2 + 2x - 3$

i) when $x=0$, $y=-3$

When $f(x)=0$, $(x+3)(x-1)=0$

$\therefore x = -3, 1$

$\therefore$ Intercepts at $(0,-3), (-3,0), (1,0)$

ii) vertex at $x = -\frac{b}{2a}$

$\therefore x = \frac{-2}{2}$

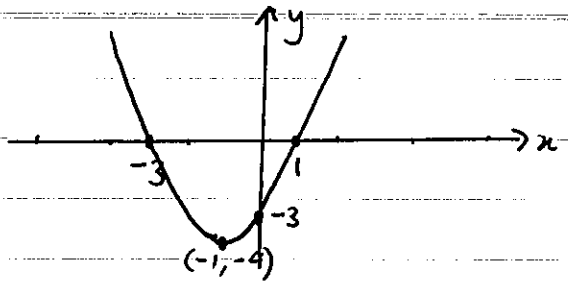
$\therefore x = -1$

When $x=-1$, $f(x) = (-1)^2 + 2(-1) - 3$

$\therefore f(x) = -4$

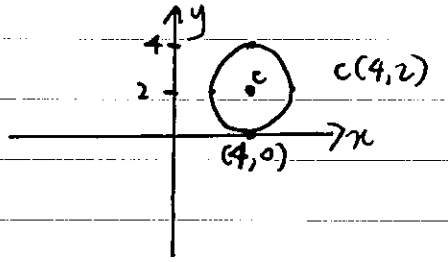
$\therefore$ minimum value is -4

iii)



iv) curve decreasing when $x < -1$

e) Locus is a circle with $C(4,2)$ and radius 2 units



f) $f(x) = x^2 + 3x - 10$

i) $f'(x) = 2x + 3$

ii) $f'(x) = 0$ when $2x + 3 = 0$

$\therefore x = -\frac{3}{2}$

Question 10

a) $\tan 300^\circ$

$= -\tan 60^\circ$

$= -\sqrt{3}$

b) $x = \frac{4 \pm \sqrt{16+96}}{6}$

$b = -4$

$a = 3$

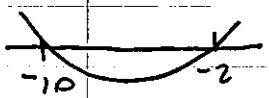
$-4ac = 96$

$\therefore -12c = 96$

$\therefore c = -8$

$\therefore$ eqn: $3x^2 - 4x - 8 = 0$

c) i) $(x+10)(x+2) \geq 0$



By inspection, $x \leq -10, x \geq -2$

ii) $\sqrt{x^2+12x+20}$

need domain $x^2+12x+20 \geq 0$

$\therefore \{x : x \leq -10, x \geq -2\}$

d) i) D has y co-ordinate $y = 7$

D lies on AD

$\therefore$ lies on $2x - y - 3 = 0$

$\therefore 2x - 7 - 3 = 0$

$2x - 10 = 0$

$x = 5$

$\therefore$ D is the point $(5, 7)$

ii) $m_{BC} = \frac{7-3}{2-0}$

$= \frac{4}{2}$

$= 2$

$m_{AD} = \frac{7+3}{5-0}$

$= \frac{10}{5}$

$= 2$

$\therefore m_{BC} = m_{AD}$

$\therefore BC \parallel AD$

$\therefore ABCD$ is a trapezium

iii) $d_{BC} = \sqrt{(7-3)^2 + (2-0)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

$d_{AD} = \sqrt{(5-0)^2 + (7+3)^2}$

$= \sqrt{25 + 100}$

$= \sqrt{125}$

$= 5\sqrt{5}$

iv) $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

$= \left| \frac{2(0) - 1(3) - 3}{\sqrt{(2)^2 + (-1)^2}} \right|$

$= \left| \frac{-6}{\sqrt{5}} \right|$

$= \frac{6}{\sqrt{5}}$ units as required

v) $A = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(2\sqrt{5} + 5\sqrt{5})\frac{6}{\sqrt{5}}$

$= \frac{21\sqrt{5}}{\sqrt{5}}$

$= 21$ units^2

Question 11

a) $\frac{m^2-4}{mn} \times \frac{2m}{2m-4}$

$\frac{(m-2)(m+2)}{mn} \times \frac{2m}{2(m-2)}$

$= \frac{m+2}{n}$

b) $x^2 + 6x + 11$

$\Delta = (6)^2 - 4(1)(11)$

$= 36 - 44$

$= -12$

$\therefore \Delta < 0$

$\therefore$ no real roots

$\therefore$ doesn't cross x-axis

$\therefore$ definite

$a > 0$

$\therefore$ positive definite

c) $C(2,-3)$ $A(5,-7)$

Let B be the other point

$\therefore$ C is the midpoint of AB

$\therefore 2 = \frac{5+x_2}{2}$ $\quad -3 = \frac{-7+y_2}{2}$

$4 = 5 + x_2$ $\quad -6 = -7 + y_2$

$\therefore x_2 = -1$ $\quad \therefore y_2 = 1$

$\therefore$ B is the point $(-1, 1)$

d) $3x^2 + 5x - 1 = 0$

i) $\alpha + \beta = \frac{-5}{3}$

ii) $\alpha\beta = \frac{-1}{3}$

iii) $(\alpha+1)(\beta+1)$

$= \alpha\beta + \alpha + \beta + 1$

$= -\frac{1}{3} + \frac{-5}{3} + 1$

$= -1$

e) i) $\frac{d}{dx} x^{-3}$

$= -3x^{-4}$

$= \frac{-3}{x^4}$

ii) $\frac{d}{dx}(x-3)(2x+1)$

$= \frac{d}{dx} 2x^2 - 5x - 3$

$= 4x - 5$

iii) $\frac{d}{dx} \frac{5x^6 - 7x^2}{x}$

$= \frac{d}{dx}(5x^5 - 7x)$

$= 25x^4 - 7$

Question 12

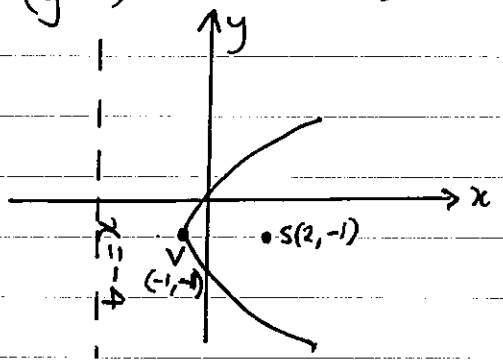
a) i) $S(2,-1)$ $\quad d: x = -4$

concave right parabola

$V(-1,-1)$

ii) $(y+1)^2 = 4(3)(x+1)$

$(y+1)^2 = 12(x+1)$



b) $(x^2-2)^2 - 4(x^2-2) - 21 = 0$

let $u = x^2 - 2$

$\therefore u^2 - 4u - 21 = 0$

$(u-7)(u+3) = 0$

$u = 7, -3$

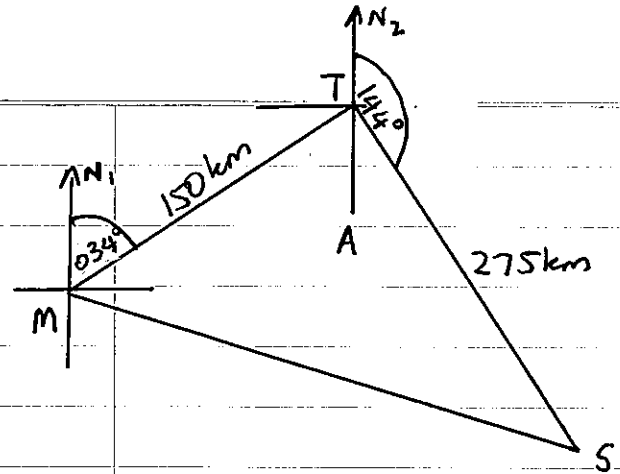
$\therefore x^2 - 2 = 7$ or $x^2 - 2 = -3$

$x^2 = 9$ $\quad x^2 = -1$

$\therefore x = \pm 3$ $\quad$ no real solutions

$\therefore x = \pm 3$

c)



i) $\angle MTA = 34°$ (alternate angles in parallel lines)

$\angle STA + 144 = 180$ (angle sum of straight angle

$\therefore \angle STA = 36$

$\therefore \angle MTS = 34 + 36$

$\therefore \angle MTS = 70°$ as required

ii) $(MS)^2 = (150)^2 + (275)^2 - 2(150)(275)\cos 70°$

$= 22500 + 75625 - 82500\cos 70°$

$= 98125 - 28216.66182$

$= 69908.33817$

$MS \doteq 264.4$ km

$\therefore MS = 264$ km (to nearest km)

iii) $\frac{\sin \angle TMS}{275} = \frac{\sin 70°}{MS}$

$\sin \angle TMS = 0.977358785$

$\angle TMS = 78°$ (to nearest degree)

Bearing $= (034° + 078°)$

$\therefore$ Bearing $= 112°$

d) $f(x) = x^2 - 1$

i) $f(2) = (2)^2 - 1$

$= 3$

$f(2+h) = (2+h)^2 - 1$

$f(2+h) = 4 + 4h + h^2 - 1$

$= h^2 + 4h + 3$

ii) $\frac{f(2+h) - f(2)}{h}$

$= \frac{h^2 + 4h + 3 - 3}{h}$

$= \frac{h(h+4)}{h}$

$= 4 + h$ as required

iii) $f'(x) = 2x$

$\therefore f'(2) = 2(2)$

$= 4$

Question 13

a) $2x^2 - 3x - 4 \equiv A(x+2)^2 + B(x+2) + C$

When $x = 0$: $2(0)^2 - 3(0) - 4 = A(0+2)^2 + B(0+2) + C$

$-4 = 4A + 2B + C$

When $x = -2$: $2(-2)^2 - 3(-2) - 4 = A(-2+2)^2 + B(-2+2) + C$

$8 + 6 - 4 = C$

$\therefore \boxed{C = 10}$

$\therefore 4A + 2B + 10 = -4$

$4A + 2B = -14$

$\boxed{2A + B = -7} \Rightarrow \boxed{B = -7 - 2A}$

When $x = 1$: $2(1)^2 - 3(1) - 4 = A(1+2)^2 + B(1+2) + C$

$-5 = 9A + 3B + 10$

$\boxed{9A + 3B = -15}$

$\therefore \boxed{3A + B = -5} \Rightarrow \boxed{B = -5 - 3A}$

$\therefore -7 - 2A = -5 - 3A$

$A = 2$

$\therefore B = -11$

$\therefore A = 2, B = -11, C = 10$

b) i) The function is not even as it doesn't have line symmetry about the y-axis
The function is not odd as it doesn't have point symmetry about the origin
∴ The function is neither odd nor even

ii) $f(x)=0$ at a, c and e

iii) $f'(x)=0$ at b and d

iv) $f'(x)>0$ for $b<x<d$

Question 13

a) i) $\sin 30° = \frac{4}{BD}$

$\frac{1}{2} = \frac{4}{BD}$

$\therefore BD = 8$ units

ii) $\angle BDC = 60°$ (angle sum $\triangle BDC$)
$\therefore \angle ADB + 60° = 180°$ (angle sum straight angle $\angle ADC$)
$\therefore \angle ADB = 120°$

$\frac{\sin x}{AD} = \frac{\sin 120°}{13}$

$\sin x = \frac{AD \cdot \sqrt{3}}{2(13)}$

$\boxed{\sin x = \frac{\sqrt{3}\,AD}{26}}$

$(BC)^2 + (DC)^2 = (BD)^2$ (by Pythagoras' Thm)
$(BC)^2 + 16 = 64$

$\therefore (BC)^2 = 48$
$\therefore BC = \sqrt{48}$
$(BC)^2 + (AC)^2 = (AB)^2$ (Pythag)
$\therefore (\sqrt{48})^2 + (AC)^2 = (13)^2$
$48 + (AC)^2 = 169$
$(AC)^2 = 121$
$\therefore AC = 11$
$\therefore AD = 11 - 4$
$\therefore AD = 7$
$\therefore \sin x = \frac{\sqrt{3} \cdot 7}{26}$
$\therefore \sin x = \frac{7\sqrt{3}}{26}$

d) i) $y = x + m$ $\quad x^2 + y^2 = 4$
intersect when $x^2 + (x+m)^2 = 4$
$x^2 + x^2 + 2mx + m^2 = 4$
$\therefore 2x^2 + 2mx + m^2 - 4 = 0$
as required

ii) line will be a tangent with one point of intersection
$\therefore$ need $\Delta = 0$
$\Delta = (2m)^2 - 4(2)(m^2-4)$
$= 4m^2 - 8m^2 + 32$
$= -4m^2 + 32$
for $\Delta = 0$, $-4m^2 + 32 = 0$
$\therefore 4m^2 = 32$
$m^2 = 8$
$m = \pm\sqrt{8}$
$\therefore m = \pm 2\sqrt{2}$