

2013 Preliminary Course FINAL EXAMINATION Friday, September 6

Mathematics

General Instructions

- Reading Time 5 minutes.
- Working Time 3 hours.
- Write using a black pen.
- Approved calculators may be used.
- All necessary working should be shown for every question.

Total marks (100)

Section I

10 marks

- \circ Attempt Questions 1 10
- Answer on the multiple choice answer sheet provided
- Allow approximately 15 minutes for this section

Section II

90 marks

- Attempt Questions 11 16
- o Answer in the booklets provided
- Begin each question in a new booklet

Student Number:

• Allow approximately 2 hours 45 minutes for this section

Name: _____

Teacher: _____

- **P2** provides reasoning to support conclusions which are appropriate to the context.
- **P3** performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.
- **P4** chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.
- **P5** understands the concept of a function and the relationship between a function and its graph.

Section I

10 marks Attempt Questions 1-10 Allow about 15 minutes for this section

Use the multiple choice answer sheet for your responses to Questions 1 - 10.

- 1 What is the value of $\frac{18.81-3.47}{2.79+7.75}$ correct to two significant figures? (A) 1.4 (B) 1.45
 - (C) 1.46
 - (D) 1.5

2 Which of the following equations has solutions x = 2 and x = -3?

- (A) $x^2 5x 6 = 0$
- (B) $x^2 + 5x 6 = 0$
- (C) $x^2 x 6 = 0$
- (D) $x^2 + x 6 = 0$

3 Which statement is incorrect?

- (A) The diagonals of a rhombus bisect each other
- (B) The diagonals of a rhombus are equal
- (C) The diagonals of a rhombus are perpendicular to each other
- (D) The diagonals of a rhombus bisect the vertex angles

4 The function $f(x) = \frac{x^2 - 1}{x}$ is:

- (A) an even function
- (B) an odd function
- (C) neither an even nor odd function
- (D) a zero function





Not to scale

(A)
$$15^2 = 16^2 + 20^2 + 2 \times 16 \times 20 \cos \theta$$

(B)
$$\cos\theta = \frac{16^2 + 20^2 - 15^2}{2 \times 20 \times 15}$$

(C)
$$\frac{15}{\sin\theta} = \frac{16}{\sin 52^{\circ}3'}$$

(D)
$$\frac{\sin\theta}{16} = \frac{\sin 52^{\circ}3^{\circ}}{15}$$

What is the gradient of the line perpendicular to the line 2x + y + 3 = 0?

7

(A)
$$-2$$

(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2

8 The solution to $t^2 > t$ is:

(A)
$$0 < t < 1$$

- (B) t < 0 or t > 1
- (C) t > 1
- (D) t < -1 or t > 0

9 What is the exact value of cos 240)°?
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(A)
$$\frac{1}{2}$$

(B) $-\frac{1}{2}$
(C) $\frac{\sqrt{3}}{2}$
(D) $-\frac{\sqrt{3}}{2}$

10 Simplify
$$\frac{\cos(90^\circ - \theta)}{\sin(90^\circ - \theta)}$$
(A) 1
(B) $\cot \theta$
(C) $\tan \theta$
(D) $-\tan \theta$

End of Section I

Section II

90 marks **Attempt Questions 11-16** Allow about 2 hours 45 minutes for this section Begin each question in a new booklet All necessary working should be shown All questions are of equal value

Ques	tion 11 (15 marks). Use a SEPARATE writing booklet.	Marks
(a)	Solve $5x - 2 = x + 10$	1
(b)	Factorise:	
	(i) $2x^2 + 7x - 4$	1
	(ii) $(x-2)^2 - 16$	2
(c)	The line $6x - ky = 2$ passes through the point (3, 2). Find the value of <i>k</i> .	2
(d)	Solve the following pair of simultaneous equations:	3
	x + y = 1 5x - 4y = 14	
(e)	Find the value of x if $\sqrt{75} + \sqrt{27} = \sqrt{x}$	3

Question 11 continues on the next page

(f) ABCD is a parallelogram. CB is produced to E so that CB = BE.



Prove $\Delta AFD \equiv \Delta EFB$

3

Question 12 (15 marks). Use a SEPARATE writing booklet.

(a) Solve
$$\frac{3}{x-2} - \frac{5}{2} = 1$$
 3

(b) Solve
$$2+3x = |x+1|$$
 3

(c) What are the values of a and b if
$$\frac{5-2\sqrt{2}}{1+\sqrt{2}} = a + b\sqrt{2}$$
? 3

(d) Prove
$$\tan \theta + \sec \theta = \frac{1 + \sin \theta}{\cos \theta}$$
 2

(e) BF is parallel to CG, BC = EC and $\angle ABE = 112^{\circ}$.



(i)	Show that $\angle BEC = 68^{\circ}$.	2

(ii) Hence, or otherwise, show that CG bisects $\angle DCE$. 2

Marks

(a) In the diagram below the points *A*, *B* and *C* have coordinates (1, -2), (-4, -3) and (-1, 3) respectively.



(i)	Calculate the exact length of interval <i>BC</i>	2
(ii)	Find the gradient of <i>BC</i>	1
(iii)	Hence, show that the equation of <i>BC</i> is $y = 2x + 5$	1
(iv)	Find, to the nearest degree, the acute angle between the <i>x</i> -axis and the line BC	2
(v)	Find the perpendicular distance between A and the line BC	2
(vi)	Find the coordinates of <i>D</i> , in the first quadrant, so that <i>ABCD</i> is a parallelogram	2
(vii)	Find the exact area of the parallelogram ABCD	1
The le $c = 6.7$	ngths of the sides of triangle <i>ABC</i> are $a = 5.2$ cm, $b = 7.3$ cm and r cm.	
(i)	Explain why $\angle BAC$ is the smallest angle in the triangle.	1
(ii)	Calculate the size of the smallest angle in $\triangle ABC$. Give the answer correct to the nearest minute.	2
(iii)	Hence find the area of the triangle. Give the answer correct to the nearest square centimetre.	1

(b)

2

2

(a) In the diagram, AOB and COD are straight lines. $AC \perp AB$ and $AB \perp BD$.



- (i) Prove $\Delta ACO \parallel \mid \Delta BDO$
- (ii) If *CD* is 35 cm, find the length of *OD*
- (b) A ship leaves port P and travels on a bearing of 104° a distance of 300 km to point Q. It then turns and travels on a bearing of 239° for 200 km to point R.



(a) The function y = f(x) is defined as follows:

$$f(x) = \begin{cases} x - 1 & \text{for } x \le -2 \\ -1 & \text{for } -2 < x < 1 \\ x + 1 & \text{for } x \ge 1 \end{cases}$$

(i) Evaluate
$$f(-2) + f(1)$$
 2

Marks

(ii) Write an expression for
$$f(a^2+1)$$
 1

(b) Make neat sketches of the following graphs on separate number planes. Mark clearly the essential features of each graph.

(i)
$$(x-3)^2 + (y+4)^2 = 25$$
 2

(ii)
$$y = 4 - x^2$$
 2

(iii)
$$xy = 2$$
 2

(iv)
$$y = 1 - 2^{-x}$$
 2

(c) Solve
$$|7-3x| < 3$$
 and graph your solution on a number line. 3

(d) If
$$\tan \theta = -\frac{4}{5}$$
 and $\cos \theta > 0$, is the value of $\sin \theta$ positive or negative? 1

(a) In the diagram below $AO \parallel BP \parallel CQ$.





(b) Graph the region represented by the inequalities:

$$x^2 + y^2 > 25$$
 and $x \le 0$

(c) Simplify
$$\frac{x^3 - 1}{x^2 - 1} \times \frac{x^2 - 4x - 5}{4x^2 + 4x + 4}$$
 3

(d) Find the domain of
$$y = \frac{1}{\sqrt{5-x}}$$
.

- (e) Find the equation of the line through the point of intersection of the lines 3 = 6x 5y = 3 and 4x + y = -11 and also through the point (2, 1)
- (f) If the points (-2a, 3), (a-1, a-2) and (a-3, a+1) are collinear, find the **3** value of *a*.

End of Examination Paper

2

2

Section I Answer Sheet

Student Number: _____

10 marks Attempt Questions 1-10 Allow about 15 minutes for this section

Use this multiple choice answer sheet for questions 1 - 10.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample

2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9 A \bigcirc B \bigcirc C \bigcirc D \bigcirc

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:



Completely fill the response oval representing the most correct answer.

1	A 🔿	B 🔿	CO	D
2	A O	BO	сO	DO
3	$A \bigcirc$	BO	СО	DO
4	$A \bigcirc$	BO	сO	DO
5	$_{\rm A}$ \bigcirc	BO	сO	DO
6	$A \bigcirc$	BO	сO	DO
7	$A \bigcirc$	BO	СО	DO
8	$A \bigcirc$	BO	СО	DO
9	$A \bigcirc$	BO	сO	DO
10	АO	вО	СО	DO

Sectio	on I Multiple Choice Solutions					
1	18.81 - 3.47	2	(x-2)(x+3) = 0			
1	2.79 + 7.75		$x^2 + x - 6 = 0$	(D)		
	=1.455					
	=1.5(2sf) (D)					
3	The diagonals of a rhombus are equal	4	$x^2 - 1$			
		4	$f(x) = \frac{1}{x}$			
	(B)		$(-x)^2 - 1$			
			$f(-x) = \frac{1}{(-x)}$			
			$r^{2}-1$			
			$=-\frac{x-1}{x}$			
			-f(x)	(B)		
			$- \int (x)$	(D)		
5	For some <i>x</i> values there are two		15 16			
	matching y values	6	$\frac{1}{\sin\theta} = \frac{1}{\sin 52^{\circ}3'}$			
	(C)			(C)		
				(0)		
7	2x + y + 3 = 0	8	$t^2 > t$			
	y = -2x - 3		$t^2 - t > 0$			
	m = -2		t(t-1) > 0			
	1		t(t-1) > 0	(D)		
	$m_{\perp} = \frac{1}{2} \tag{C}$		l < 0 of $l > 1$	(D)		
	-					
9	cos 240°	10	$\cos(90^\circ - \theta)$			
	$=\cos(180^{\circ}+60^{\circ})$	10	$\overline{\sin(90^\circ - \theta)}$			
	$=-\cos 60^{\circ}$		$\sin heta$			
	1		$=\frac{1}{\cos\theta}$			
	$=-\frac{1}{2} \tag{B}$		$= \tan \theta$	(C)		
	-					

Solutions to Year 11 Mathematics Preliminary Examination 2013

Question 11		Marking Criteria	
(a)	5x-2 = x+10 $4x = 12$ $x = 3$	1	Correct answer
(b) (i)	$2x^{2} + 7x - 4$ = (2x-1)(x+4)	1	Correct answer
(b)	$(x-2)^2 - 16$	2	Correct solution
(11)	= (x-2+4)(x-2-4) = (x+2)(x-6)	1	Attempt at difference of two squares OR Correct expansion $x^2 - 4x - 12$
(c)	Sub. (3, 2) into $6x - ky = 2$: 6(3) - k(2) = 2 2k - 16	2	Correct solution
	k = 8	1	Correct substitution of (3, 2) OR Correct answer without justification
(d)	$x + y = 1 \qquad \dots \dots (1)$ 5x - 4y - 14 (2)	3	Correct solution
	$(1) \times 5:5x + 5y = 5 \qquad \dots (3)$ (3) - (2):9y = -9 y = -1	2	Correct elimination/substitution method with only one correct value for either <i>x</i> or <i>y</i>
	In (1): $x - 1 = 1$ x = 2	1	Correct attempt at either elimination or substitution method
e)	$\sqrt{75} + \sqrt{27}$ $= 5\sqrt{3} + 3\sqrt{3}$	3	Correct solution $x = 192$
	$= 8\sqrt{3}$ $= \sqrt{64 \times 3}$	2	Substantially correct solution
	$= \sqrt{192}$ $\therefore x = 192$	1	Correct attempt at simplifying the surds

Question 11 (continued)		Ma	Marking Criteria	
(f)	E	3	Correct solution	
	$A \xrightarrow{F} B$ $D \xrightarrow{BC}$ (aquel opposite sides in perellelogram ABCD)	2	Two correct statements, justified OR Correct proof but not fully justified	
	(equal opposite sides in parallelogram ABCD) CB = BE (given) $\therefore AD = EB$ (both equal BC) $\angle AFD = \angle BFE$ (equal vertically opposite angles) $\angle DAF = \angle EBF$ (equal alternate angles, $AD \parallel EC$) $\Delta AFD \equiv \Delta EFB$ (SAA)	1	One correct statement, justified OR Correct proof without any justification	
Communication:				
d) cle f) arti	d) clear setting out and labelling of equations 1 markf) articulate reasoning, clear argument 1 mark			

Question 12		Marking Criteria	
(a)	$\frac{\frac{3}{x-2} - \frac{5}{2} = 1}{\frac{3}{x-2} - \frac{5}{2} = 1}$	3	Correct solution
	x-2 2 7x-14 = 6 7x = 20	2	Substantially correct solution
	$x = \frac{20}{7}$	1	Correct attempt at solution
(b)	2+3x = x+1 2+3x = x+1 or 2+3x = -x-1 $2x = -1 \qquad 4x = -3$ $x = -\frac{1}{2} \qquad x = -\frac{3}{4}$	3	Correct solution
	Solutions are valid if: $2+3x \ge 0$ i.e. $x \ge -\frac{2}{3}$	2	One mistake in the solution
	$\therefore x = -\frac{1}{2}$ is only valid solution OR Check solutions by substitution into equation	1	Correct attempt at solution
(c)	$\frac{5-2\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$	3	Correct solution
	$=\frac{5-5\sqrt{2}-2\sqrt{2}+4}{1-2}$ $=\frac{9-7\sqrt{2}}{1-2}$	2	One mistake in working towards the answer
	$= -9 + 7\sqrt{2}$ $\therefore a = -9 \text{ and } b = 7$	1	Correctly attempting to rationalise the denominator
(d)	$\tan \theta + \sec \theta = \frac{1 + \sin \theta}{\cos \theta}$ LHS = $\tan \theta + \sec \theta$	2	Correct solution with correct setting out
	$= \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}$ $= \frac{\sin \theta + 1}{\cos \theta}$ $= RHS$ OR starting from RHS show it equals LHS	1	Correct attempt at proof by rewriting expression in terms of $\sin \theta$ and/or $\cos \theta$ OR equivalent

Que	stion 12 (continued)	Ma	Marking Criteria	
(e) (i)	$A \xrightarrow{B} C \xrightarrow{D}$	2	Correct proof	
	Not to scale E F $CBE = 68^{\circ}$ (supplement of $\angle ABE$) $\angle CBE = \angle BEC = 68^{\circ}$ (angles opposite equal sides in isosceles $\triangle CBE$)	1	One correct statement, justified OR Correct proof, not fully justified	
(ii)	$\angle CBE = \angle DCG = 68^{\circ}$ (equal corresponding angles, <i>BF</i> <i>CG</i>)	2	Correct proof	
	$\angle BEC = \angle ECG = 68^{\circ}$ (equal alternate angles, $BF \parallel CG$) CG bisects $\angle DCE (\angle ECG = \angle GCD)$	1	One correct statement, justified OR Correct proof, not fully justified	
Con (e) c	Communication:(e) clear, concise and correct setting out of proofs4 marks			

Question 13		Marking Criteria	
(a) (i)	(-1,3)C	2	Correct answer
	$BC = \sqrt{3^2 + 6^2}$ $= \sqrt{45}$ $= 3\sqrt{5} \text{ units}$	1	Correct length as an unsimplified surd
(ii)	$m = \frac{6}{3}$ $= 2$	1	Correct answer
(iii)	y-3 = 2(x+1) y-3 = 2x+2 y = 2x+5	1	Correct solution
(iv)	$\tan \theta = 2$ $\theta = \tan^{-1}(2)$ = 63.434	2	Correct solution
	$= 63^{\circ}$ (nearest deg)	1	Correct statement $\tan \theta = 2$
(v)	$d = \frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$ $= \frac{ 2(1) - 1(-2) + 5 }{\sqrt{a^2 + b^2}}$	2	Correct solution
	$\sqrt{2^2 + 1^2}$ $= \frac{9}{\sqrt{5}} \text{ units}$	1	Correct substitution into correct formula
(vi)	$B \rightarrow A$: Right 5, up 1 $C \rightarrow D$: Right 5, up 1 ($CD = BA, CD \parallel BA$) D = (-1+5, 3+1)	2	Correct answer
	= (4, 4)	1	One correct coordinate

Ques	tion 13 (continued)	Ma	arking Criteria
(vii)	A = bh = $3\sqrt{5} \times \frac{9}{\sqrt{5}}$ = 27 units ²	1	Correct answer
(b) (i)	$\angle BAC$ is opposite the shortest side	1	Correct reason
(ii)	$\cos \angle BAC = \frac{7.3^2 + 6.7^2 - 5.2^2}{2 \times 7.3 \times 6.7}$ $= 0.72725$	2	Correct solution rounded correctly
	$\angle BAC = \cos^{-1}(0.72725)$ = 43°20'35.92" = 43°21' (nearest min)	1	Correct substitution into cosine rule OR Substantially correct solution
(iii)	$A = \frac{1}{2}bc \sin A$ = $\frac{1}{2} \times 7.3 \times 6.7 \sin 43^{\circ} 20'35''$ = 16.785 = 17 cm ² (nearest cm ²) OR $A = \frac{1}{2} \times 7.3 \times 6.7 \sin 43^{\circ} 21'$ = 16.787 = 17 cm ² (nearest cm ²)	1	Correct solution
Communication: a) v) showing substitution 1 mark vi) explaining how coordinates are found 1 mark			
b) ii) showing unrounded answer in degrees/minutes/seconds before rounding 1 mark			

Question 14		Ma	Marking Criteria	
(a) (i)	$\begin{array}{c} A \\ A \\ C \\$	2	Correct solution	
	$\angle A = \angle B = 90^{\circ} \text{ (given)}$ $\angle AOC = \angle BOD \text{ (equal vertically opposite angles)}$ $\Delta ACO \parallel \Delta BDO \text{ (equiangular)}$	1	One correct statement, justified OR Correct proof, not fully justified	
(ii)	$\frac{AC}{BD} = \frac{CO}{DO} = \frac{2}{5}$ (corresponding sides in similar)	2	Correct solution, justified	
	(concesponding sides in similar triangles are in same ratio) $OD = \frac{5}{7} \times 35$ = 25 cm	1	Correct solution, not fully justified OR Correct first statement with reason	
(b) (i)	North S North P 104° North Q 239° Not to scale 200 km	2	Correct proof	
	\swarrow_{R} $\angle PQT = 76^{\circ}$ (supplementary cointerior angles on parallel lines) $\angle TQR = 121^{\circ}$ (angles at point add to 360°) $\angle PQR = 45^{\circ} (\angle TQR - \angle PQT)$	1	Correctly finding $\angle PQT$ or $\angle TQR$, justified OR Correct proof, not fully justified	
(ii)	$RP^{2} = 300^{2} + 200^{2} - 2 \times 300 \times 200 \cos 45^{\circ}$	2	Correct solution	
	= 45147.18 $RP = \sqrt{45147.18}$ = 212.47 = 212 km (nearest km)	1	Correct substitution in cosine rule	

Question 14 (continued)		Ma	Marking Criteria	
(iii)	iii) $\frac{\sin \angle QPR}{200} = \frac{\sin 45^{\circ}}{RP}$ $\sin \angle QPR = \frac{200 \sin 45^{\circ}}{RP}$ $= 0.665$ $\angle QPR = 41.726$	3	Correct solution	
		2	Correct value of $\angle QPR$	
Bearing of R from P = $104^\circ + 42^\circ$ = 147°	Bearing of R from P = $104^\circ + 42^\circ$ = 147°	1	Correct substitution into sine rule	
(c) (i)	$2\cos^{2} x - 2 + 3\sin^{2} x = \sin^{2} x$ LHS = $2\cos^{2} x - 2 + 3\sin^{2} x$	2	Correct proof	
	$= 2(1-\sin^2 x) - 2 + 3\sin^2 x$ $= 2 - 2\sin^2 x - 2 + 3\sin^2 x$ $= \sin^2 x$ $= RHS$	1	Correct attempt at proof	
(ii)	$2\cos^{2} x - 2 + 3\sin^{2} x = 1$ $\sin^{2} x = 1$ $\sin x = \pm 1$ $x = 90^{\circ}, 270^{\circ}$	2	Correct solution	
		1	One correct value for <i>x</i>	
 Communication: (a)(i) 1 mark for copying diagram 1 mark for stating vertices in corresponding order 1 mark for sufficient proof i.e. equiangular when two pairs of corresponding angles are equal (ii) 1 mark for correct reason why corresponding pairs of sides are in same ratio 1 mark for correct units for <i>OD</i> (b)(i) 2 marks for each of correct geometric reasons to show angle <i>PQR</i> = 45° (ii) 1 mark for correct rounding (iii) 1 mark for correct rounding 				

Question 15		Marking Criteria	
(a) (i)	f(-2) + f(1) = (-2 - 1) + (1 + 1)	2	Correct solution
	=-1	1	Correct attempt at solution
(ii)	$f(a^{2}+1) = a^{2}+1+1$ = a ² +2	1	Correct answer
(b) (i)	y 4 4	2	Correct graph showing all essential features
	$\begin{array}{c} 2 \\ \hline \\ -4 \\ -2 \\ -2 \\ -4 \\ -6 \\ -8 \\ -10 \\ \end{array} $	1	Correct graph without all essential features shown
(ii)	$ \begin{array}{c} & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & $	2	Correct graph showing all essential features
		1	Correct graph without all essential features shown





Question 16 (continued)		Ma	Marking Guidelines	
(d)	$y = \frac{1}{\sqrt{5-x}}, x \neq 5$	2	Correct answer	
	Domain:			
	5 - x > 0	1	Correct attempt at finding	
	<i>x</i> < 5	1	domain by stating $x \neq 5$ or $x \leq 5$	
(e)	6x - 5y - 3 + k(4x + y + 11) = 0			
	Sub. (2, 1):	3	Correct solution	
	12 - 5 - 3 + k(8 + 1 + 11) = 0		Correct value of k	
	4 + 20k = 0		OR	
	k = -4	2	Correct point of intersection	
	$\kappa = \frac{1}{20}$	-	and attempt to find equation	
	$k = -\frac{1}{2}$		or required line	
	5		Correct form of equation	
	$6x-5y-3-\frac{1}{2}(4x+y+11)=0$		shown by first line and	
	5		attempt to substitute (2, 1)	
	30x - 25y - 15 - 4x - y - 11 = 0	1	OR Correct attempt to colum	
	20x - 20y - 20 = 0		equations simultaneously	
	x - y - 1 = 0			
(f)	(a-2)-3 = (a+1)-3			
	(a-1)+2a $(a-3)+2a$	3	Correct solution	
	$\frac{a-5}{a-2} = \frac{a-2}{a-2}$			
	3a-1 $3a-3$			
	(3a-3)(a-5) = (3a-1)(a-2)	2	Substantially correct solution	
	$3a^2 - 18a + 15 = 3a^2 - 7a + 2$			
	11a = 13			
	$a = \frac{13}{11}$		Correct attempt at solution	
	11	1	shown by first line of working	
Communication: Question 16 (4 marks)				
(b) The graph is drawn neatly with template Intercents shown and graph labelled 1 Mark				
(b) The graph is drawn neatly with template. Intercepts shown and graph labelled. I Mark				

(d) Clear setting out. For example stating limitations such as $x \neq 5$. **1 Mark**

- (e) Clear and logical setting out of solution (example: mention of substitution). **1 Mark**
- (f) Clear explanation that equal gradients will prove the points are collinear. **1 Mark**