

## 2013

## Preliminary Course

FINAL EXAMINATION
Friday, September 6

## Mathematics

## General Instructions

o Reading Time - 5 minutes.
o Working Time - 3 hours.
o Write using a black pen.
o Approved calculators may be used.
o All necessary working should be shown for every question.

## Total marks (100)

## Section I

## 10 marks

o Attempt Questions 1 - 10
o Answer on the multiple choice answer sheet provided
o Allow approximately 15 minutes for this section

## Section II

90 marks
o Attempt Questions 11 - 16
o Answer in the booklets provided
o Begin each question in a new booklet
o Allow approximately 2 hours 45 minutes for this section

Student Number: $\qquad$

Teacher: $\qquad$

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.
provides reasoning to support conclusions which are appropriate to the context.
P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.
understands the concept of a function and the relationship between a function and its graph.

## Section I

## 10 marks

## Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for your responses to Questions 1 - 10 .

1 What is the value of $\frac{18.81-3.47}{2.79+7.75}$ correct to two significant figures?
(A) 1.4
(B) 1.45
(C) 1.46
(D) 1.5

2 Which of the following equations has solutions $x=2$ and $x=-3$ ?
(A) $x^{2}-5 x-6=0$
(B) $x^{2}+5 x-6=0$
(C) $x^{2}-x-6=0$
(D) $x^{2}+x-6=0$

3 Which statement is incorrect?
(A) The diagonals of a rhombus bisect each other
(B) The diagonals of a rhombus are equal
(C) The diagonals of a rhombus are perpendicular to each other
(D) The diagonals of a rhombus bisect the vertex angles

4 The function $f(x)=\frac{x^{2}-1}{x}$ is:
(A) an even function
(B) an odd function
(C) neither an even nor odd function
(D) a zero function

5 Which graph does not represent a function?
(A)

(B)

(C)

(D)


6 Which of the following is a correct expression involving $\theta$ in triangle $A B C$ ?


Not to scale
(A) $15^{2}=16^{2}+20^{2}+2 \times 16 \times 20 \cos \theta$
(B) $\cos \theta=\frac{16^{2}+20^{2}-15^{2}}{2 \times 20 \times 15}$
(C) $\frac{15}{\sin \theta}=\frac{16}{\sin 52^{\circ} 3^{\prime}}$
(D) $\frac{\sin \theta}{16}=\frac{\sin 52^{\circ} 3^{\prime}}{15}$

7 What is the gradient of the line perpendicular to the line $2 x+y+3=0$ ?
(A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2

8 The solution to $t^{2}>t$ is:
(A) $0<t<1$
(B) $t<0$ or $t>1$
(C) $t>1$
(D) $t<-1$ or $t>0$
$9 \quad$ What is the exact value of $\cos 240^{\circ}$ ?
(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) $\frac{\sqrt{3}}{2}$
(D) $-\frac{\sqrt{3}}{2}$

10 Simplify $\frac{\cos \left(90^{\circ}-\theta\right)}{\sin \left(90^{\circ}-\theta\right)}$
(A) 1
(B) $\cot \theta$
(C) $\tan \theta$
(D) $-\tan \theta$

## Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours 45 minutes for this section
Begin each question in a new booklet
All necessary working should be shown
All questions are of equal value

Question 11 (15 marks). Use a SEPARATE writing booklet.
Marks
(a) Solve $5 x-2=x+10$

1
(b) Factorise:
(i) $2 x^{2}+7 x-4 \quad 1$
(ii) $(x-2)^{2}-16 \quad 2$
(c) The line $6 x-k y=2$ passes through the point (3, 2). Find the value of $k$.
(d) Solve the following pair of simultaneous equations:

$$
\begin{gathered}
x+y=1 \\
5 x-4 y=14
\end{gathered}
$$

(e) Find the value of $x$ if $\sqrt{75}+\sqrt{27}=\sqrt{x}$
(f) $\quad A B C D$ is a parallelogram. $C B$ is produced to $E$ so that $C B=B E$.


Prove $\triangle A F D \equiv \triangle E F B$

Question 12 (15 marks). Use a SEPARATE writing booklet.
(a) Solve $\frac{3}{x-2}-\frac{5}{2}=1$
(b) Solve $2+3 x=|x+1|$
(c) What are the values of $a$ and $b$ if $\frac{5-2 \sqrt{2}}{1+\sqrt{2}}=a+b \sqrt{2}$ ?
(d) Prove $\tan \theta+\sec \theta=\frac{1+\sin \theta}{\cos \theta}$
(e) $B F$ is parallel to $C G, B C=E C$ and $\angle A B E=112^{\circ}$.


Not to scale
(i) Show that $\angle B E C=68^{\circ}$.

2
(ii) Hence, or otherwise, show that $C G$ bisects $\angle D C E$.

Question 13 (15 marks). Use a SEPARATE writing booklet.
(a) In the diagram below the points $A, B$ and $C$ have coordinates (1, -2 ), ( $-4,-3$ ) and $(-1,3)$ respectively.

(i) Calculate the exact length of interval $B C \quad 2$
(ii) Find the gradient of $B C$
(iii) Hence, show that the equation of $B C$ is $y=2 x+5$
(iv) Find, to the nearest degree, the acute angle between the $x$-axis and the line $B C$
(v) Find the perpendicular distance between $A$ and the line $B C$

2
(vi) Find the coordinates of $D$, in the first quadrant, so that $A B C D$ is a parallelogram
(vii) Find the exact area of the parallelogram $A B C D$
(b) The lengths of the sides of triangle $A B C$ are $a=5.2 \mathrm{~cm}, b=7.3 \mathrm{~cm}$ and $c=6.7 \mathrm{~cm}$.
(i) Explain why $\angle B A C$ is the smallest angle in the triangle.
(ii) Calculate the size of the smallest angle in $\triangle A B C$. Give the answer correct to the nearest minute.
(iii) Hence find the area of the triangle. Give the answer correct to the 1 nearest square centimetre.

Question 14 (15 marks). Use a SEPARATE writing booklet.
(a) In the diagram, $A O B$ and $C O D$ are straight lines. $A C \perp A B$ and $A B \perp B D$.

(i) Prove $\triangle A C O \| \Delta B D O \quad 2$
(ii) If $C D$ is 35 cm , find the length of $O D$
(b) A ship leaves port $P$ and travels on a bearing of $104^{\circ}$ a distance of 300 km to point $Q$. It then turns and travels on a bearing of $239^{\circ}$ for 200 km to point $R$.

(i) Show that $\angle P Q R=45^{\circ}$.
(ii) What is the distance from $R$ to $P$ ?

Answer correct to the nearest kilometre.
(iii) Find the bearing of $R$ from $P$ ?

Answer correct to the nearest degree.
(c) (i) Show that $2 \cos ^{2} x-2+3 \sin ^{2} x=\sin ^{2} x$
(ii) Hence, or otherwise, solve $2 \cos ^{2} x-2+3 \sin ^{2} x=1$ for $0^{\circ} \leq x \leq 360^{\circ}$

Question 15 (15 marks). Use a SEPARATE writing booklet.
(a) The function $y=f(x)$ is defined as follows:

$$
f(x)= \begin{cases}x-1 & \text { for } x \leq-2 \\ -1 & \text { for }-2<x<1 \\ x+1 & \text { for } x \geq 1\end{cases}
$$

(i) Evaluate $f(-2)+f(1) \quad 2$
(ii) Write an expression for $f\left(a^{2}+1\right)$
(b) Make neat sketches of the following graphs on separate number planes. Mark clearly the essential features of each graph.
(i) $\quad(x-3)^{2}+(y+4)^{2}=25 \quad 2$
(ii) $y=4-x^{2} \quad 2$
(iii) $x y=2 \quad 2$
(iv) $y=1-2^{-x} \quad 2$
(c) Solve $|7-3 x|<3$ and graph your solution on a number line.
(d) If $\tan \theta=-\frac{4}{5}$ and $\cos \theta>0$, is the value of $\sin \theta$ positive or negative?

Question 16 (15 marks). Use a SEPARATE writing booklet.
(a) In the diagram below $A O\|B P\| C Q$.


Find the value of $x$
(b) Graph the region represented by the inequalities:

$$
x^{2}+y^{2}>25 \text { and } x \leq 0
$$

(c) Simplify $\frac{x^{3}-1}{x^{2}-1} \times \frac{x^{2}-4 x-5}{4 x^{2}+4 x+4}$
(d) Find the domain of $y=\frac{1}{\sqrt{5-x}}$.
(e) Find the equation of the line through the point of intersection of the lines
$6 x-5 y=3$ and $4 x+y=-11$ and also through the point $(2,1)$
(f) If the points $(-2 a, 3),(a-1, a-2)$ and $(a-3, a+1)$ are collinear, find the value of $a$.

## Section I Answer Sheet

## Student Number:

$\qquad$

## 10 marks

## Attempt Questions 1-10

Allow about 15 minutes for this section

Use this multiple choice answer sheet for questions 1 - 10 .
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

## Sample

$2+4=$ ?
(A) 2
(B) 6
(C) 8
(D) 9
A
B
C $\bigcirc$
D $\bigcirc$

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.
A

C
D $\bigcirc$

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word correct and drawing an arrow as follows:


Completely fill the response oval representing the most correct answer.

| 1 | A $\bigcirc$ | B $\bigcirc$ | $\mathrm{C} \bigcirc$ |
| :---: | :---: | :---: | :---: |
| 2 | A $\bigcirc$ | B | $\mathrm{C} \bigcirc$ |
| 3 | A $\bigcirc$ | B $\bigcirc$ | $\mathrm{C} \bigcirc$ |
| 4 | A $\bigcirc$ | B $\bigcirc$ | $\mathrm{C} \bigcirc$ |
| 5 | A $\bigcirc$ | B $\bigcirc$ | $\mathrm{C} \bigcirc$ |
| 6 | A $\bigcirc$ | B $\bigcirc$ | $\mathrm{C} \bigcirc$ |
| 7 | A $\bigcirc$ | B $\bigcirc$ | $\mathrm{C} \bigcirc$ |
| 8 | A $\bigcirc$ | B $\bigcirc$ | $\mathrm{C} \bigcirc$ |
| 9 | A $\bigcirc$ | B $\bigcirc$ | $\mathrm{C} \bigcirc$ |
| 10 | A $\bigcirc$ | $\mathrm{B} \bigcirc$ | $\mathrm{C} \bigcirc$ |

Solutions to Year 11 Mathematics Preliminary Examination 2013

| Section I Multiple Choice Solutions |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\begin{align*} & \frac{18.81-3.47}{2.79+7.75} \\ & =1.455 \ldots \\ & =1.5(2 \mathrm{sf}) \tag{D} \end{align*}$ | 2 | $\begin{aligned} & (x-2)(x+3)=0 \\ & x^{2}+x-6=0 \end{aligned}$ | (D) |
|  | The diagonals of a rhombus are equal <br> (B) | $4$ | $\begin{aligned} f(x) & =\frac{x^{2}-1}{x} \\ f(-x) & =\frac{(-x)^{2}-1}{(-x)} \\ & =-\frac{x^{2}-1}{x} \\ & =-f(x) \end{aligned}$ |  |
| 5 | For some $x$ values there are two matching $y$ values |  | $\frac{15}{\sin \theta}=\frac{16}{\sin 52^{\circ} 3^{\prime}}$ | (C) |
|  | $\begin{align*} & 2 x+y+3=0 \\ & y=-2 x-3 \\ & m=-2 \\ & m_{\perp}=\frac{1}{2} \tag{C} \end{align*}$ | 8 | $\begin{aligned} & t^{2}>t \\ & t^{2}-t>0 \\ & t(t-1)>0 \\ & t<0 \text { or } t>1 \end{aligned}$ |  |
|  | $\begin{align*} & \cos 240^{\circ} \\ & =\cos \left(180^{\circ}+60^{\circ}\right) \\ & =-\cos 60^{\circ} \\ & =-\frac{1}{2} \tag{B} \end{align*}$ | 10 | $\begin{aligned} & \frac{\cos \left(90^{\circ}-\theta\right)}{\sin \left(90^{\circ}-\theta\right)} \\ & =\frac{\sin \theta}{\cos \theta} \\ & =\tan \theta \end{aligned}$ |  |


| Question 11 |  | Marking Criteria |  |
| :---: | :---: | :---: | :---: |
| (a) | $\begin{aligned} 5 x-2 & =x+10 \\ 4 x & =12 \\ x & =3 \end{aligned}$ | 1 | Correct answer |
| (b) <br> (i) | $\begin{aligned} & 2 x^{2}+7 x-4 \\ & =(2 x-1)(x+4) \end{aligned}$ | 1 | Correct answer |
| (b) <br> (ii) | $\begin{aligned} & (x-2)^{2}-16 \\ & =(x-2+4)(x-2-4) \\ & =(x+2)(x-6) \end{aligned}$ | 2 | Correct solution <br> Attempt at difference of two squares <br> OR <br> Correct expansion $x^{2}-4 x-12$ |
| (c) | Sub. $(3,2)$ into $6 x-k y=2$ : $\begin{aligned} & 6(3)-k(2)=2 \\ & 2 k=16 \\ & k=8 \end{aligned}$ | 2 1 | Correct solution <br> Correct substitution of $(3,2)$ OR <br> Correct answer without justification |
| (d) | $\begin{align*} & x+y=1  \tag{1}\\ & 5 x-4 y=14 \tag{2} \end{align*}$ <br> (1) $\times 5: 5 x+5 y=5$ <br> (3) $-(2): 9 y=-9$ $y=-1$ <br> In (1) : $x-1=1$ $x=2$ | 3 2 | Correct solution <br> Correct elimination/substitution method with only one correct value for either $x$ or $y$ |
|  |  | 1 | Correct attempt at either elimination or substitution method |
| e) | $\begin{aligned} & \sqrt{75}+\sqrt{27} \\ & =5 \sqrt{3}+3 \sqrt{3} \\ & =8 \sqrt{3} \\ & =\sqrt{64 \times 3} \\ & =\sqrt{192} \\ & \therefore x=192 \end{aligned}$ | 3 | Correct solution $x=192$ |
|  |  | 2 | Substantially correct solution |
|  |  | 1 | Correct attempt at simplifying the surds |



| Question 12 |  | Marking Criteria |  |
| :---: | :---: | :---: | :---: |
| (a) | $\frac{3}{x-2}-\frac{5}{2}=1$ $3 \quad 7$ | 3 | Correct solution |
|  | $7 x-14=6$ | 2 | Substantially correct solution |
|  | $x=\frac{20}{7}$ | 1 | Correct attempt at solution |
| (b) |  | 3 | Correct solution |
|  | $2+3 x \geq 0$ <br> i.e. $x \geq-\frac{2}{3}$ | 2 | One mistake in the solution |
|  | $\therefore x=-\frac{1}{2}$ is only valid solution <br> OR <br> Check solutions by substitution into equation | 1 | Correct attempt at solution |
| (c) | $\frac{5-2 \sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$ | 3 | Correct solution |
|  | $\begin{aligned} & \frac{1-2}{9-7 \sqrt{2}} \end{aligned}$ | 2 | One mistake in working towards the answer |
|  | $\begin{aligned} & =-9+7 \sqrt{2} \\ & \therefore a=-9 \text { and } b=7 \end{aligned}$ | 1 | Correctly attempting to rationalise the denominator |
| (d) | $\tan \theta+\sec \theta=\frac{1+\sin \theta}{\cos \theta}$ $\text { LHS }=\tan \theta+\sec \theta$ | 2 | Correct solution with correct setting out |
|  | $\begin{aligned} & =\frac{1 \cos \theta}{\cos }+\frac{1}{\cos \theta} \\ & =\frac{\sin \theta+1}{\cos \theta} \\ & =\text { RHS } \end{aligned}$ <br> OR starting from RHS show it equals LHS | 1 | Correct attempt at proof by rewriting expression in terms of $\sin \theta$ and/or $\cos \theta$ OR equivalent |


| Question 12 (continued) |  | Marking Criteria |  |
| :---: | :---: | :---: | :---: |
| (e)(i) | A $\qquad$ B <br> C <br> D <br> $112^{\circ}$ <br>  | 2 | Correct proof |
|  | $\begin{aligned} & \left.\angle C B E=68^{\circ} \text { (supplement of } \angle A B E\right) \\ & \angle C B E=\angle B E C=68^{\circ} \end{aligned}$ <br> (angles opposite equal sides in isosceles $\triangle C B E$ ) | 1 | One correct statement, justified <br> OR <br> Correct proof, not fully justified |
| (ii) | $\angle C B E=\angle D C G=68^{\circ}$ <br> (equal corresponding angles, $B F \\| C G$ ) | 2 | Correct proof |
|  | $\angle B E C=\angle E C G=68^{\circ}$ <br> (equal alternate angles, $B F \\| C G$ ) $C G$ bisects $\angle D C E(\angle E C G=\angle G C D)$ | 1 | One correct statement, justified <br> OR <br> Correct proof, not fully justified |
| Communication: <br> (e) clear, concise and correct setting out of proofs |  | mar |  |


| Question 13 |  |
| :--- | :--- | :--- | :--- | :--- |
| (a) |  |
| (i) |  |



| Question 14 |  |
| :--- | :--- | :--- | :--- |
| (a) |  |
| (i) |  |


| Question 14 (continued) |  |  | Marking Criteria |
| :---: | :---: | :---: | :---: |
| (iii) | $\begin{aligned} & \frac{\sin \angle Q P R}{200}=\frac{\sin 45^{\circ}}{R P} \\ & \sin \angle O P R=200 \sin 45^{\circ} \end{aligned}$ | 3 | Correct solution |
|  | $\begin{aligned} & =0.665 \ldots \\ & \angle Q P R=41.726 \ldots \end{aligned}$ | 2 | Correct value of $\angle Q P R$ |
|  | $\begin{aligned} & =104^{\circ}+42^{\circ} \\ & =147^{\circ} \end{aligned}$ | 1 | Correct substitution into sine rule |
| (c) <br> (i) | $\begin{aligned} & 2 \cos ^{2} x-2+3 \sin ^{2} x=\sin ^{2} x \\ & \begin{aligned} \text { LHS } & =2 \cos ^{2} x-2+3 \sin ^{2} x \\ & =2\left(1-\sin ^{2} x\right)-2+3 \sin ^{2} x \\ & =2-2 \sin ^{2} x-2+3 \sin ^{2} x \\ & =\sin ^{2} x \\ & =\text { RHS } \end{aligned} \end{aligned}$ | 2 | Correct proof |
|  |  | 1 | Correct attempt at proof |
| (ii) | $\begin{aligned} & 2 \cos ^{2} x-2+3 \sin ^{2} x=1 \\ & \sin ^{2} x=1 \\ & \sin x= \pm 1 \\ & x=90^{\circ}, 270^{\circ} \end{aligned}$ | 2 | Correct solution |
|  |  | 1 | One correct value for $x$ |
| Communication: <br> (a)(i) 1 mark for copying diagram 1 mark for stating vertices in corresponding order 1 mark for sufficient proof i.e. equiangular when two pairs of corresponding angles are equal <br> (ii) 1 mark for correct reason why corresponding pairs of sides are in same ratio 1 mark for correct units for $O D$ <br> (b)(i) 2 marks for each of correct geometric reasons to show angle $P Q R=45^{\circ}$ <br> (ii) 1 mark for correct rounding <br> (iii) 1 mark for correct rounding |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |


| Question 15 |  | Marking Criteria |  |
| :---: | :---: | :---: | :---: |
| (a) <br> (i) | $\begin{aligned} & f(-2)+f(1) \\ = & (-2-1)+(1+1) \\ = & -1 \end{aligned}$ | 2 | Correct solution |
|  |  | 1 | Correct attempt at solution |
| (ii) | $\begin{aligned} f\left(a^{2}+1\right) & =a^{2}+1+1 \\ & =a^{2}+2 \end{aligned}$ | 1 | Correct answer |
| (b)(i) |  | 2 | Correct graph showing all essential features |
|  |  | 1 | Correct graph without all essential features shown |
| (ii) |  | 2 | Correct graph showing all essential features |
|  |  | 1 | Correct graph without all essential features shown |




| Question 16 (continued) |  | Marking Guidelines |  |
| :---: | :---: | :---: | :---: |
| (d) | $y=\frac{1}{\sqrt{5-x}}, x \neq 5$ <br> Domain: $\begin{aligned} & 5-x>0 \\ & x<5 \end{aligned}$ | 2 | Correct answer |
|  |  | 1 | Correct attempt at finding domain by stating $x \neq 5$ or $x \leq 5$ |
| (e) | $6 x-5 y-3+k(4 x+y+11)=0$ <br> Sub. $(2,1)$ : $\begin{aligned} 12-5-3+k(8+1+11) & =0 \\ 4+20 k & =0 \\ k & =\frac{-4}{20} \\ k & =-\frac{1}{5} \end{aligned}$ $\begin{aligned} 6 x-5 y-3-\frac{1}{5}(4 x+y+11) & =0 \\ 30 x-25 y-15-4 x-y-11 & =0 \\ 26 x-26 y-26 & =0 \\ x-y-1 & =0 \end{aligned}$ $\begin{aligned} \frac{(a-2)-3}{(a-1)+2 a} & =\frac{(a+1)-3}{(a-3)+2 a} \\ \frac{a-5}{3 a-1} & =\frac{a-2}{3 a-3} \\ (3 a-3)(a-5) & =(3 a-1)(a-2) \\ 3 a^{2}-18 a+15 & =3 a^{2}-7 a+2 \\ 11 a & =13 \\ a & =\frac{13}{11} \end{aligned}$ | 3 | Correct solution |
|  |  | 2 | Correct value of $k$ OR <br> Correct point of intersection and attempt to find equation of required line |
|  |  | 1 | Correct form of equation shown by first line and attempt to substitute $(2,1)$ OR <br> Correct attempt to solve equations simultaneously |
| (f) |  | 3 | Correct solution |
|  |  | 2 | Substantially correct solution |
|  |  | 1 | Correct attempt at solution shown by first line of working |

## Communication: Question 16 (4 marks)

(b) The graph is drawn neatly with template. Intercepts shown and graph labelled. 1 Mark
(d) Clear setting out. For example stating limitations such as $x \neq 5$. $\mathbf{1}$ Mark
(e) Clear and logical setting out of solution (example: mention of substitution). 1 Mark
(f) Clear explanation that equal gradients will prove the points are collinear. $\mathbf{1}$ Mark

