Caringbah High School

Student Name

2013 Year 11
Semester 1Examination

Mathematics (2 Unit)

General Instructions

- Approved calculators permitted.
- Start each question in a new booklet.
- Answers without working, mathematical reasoning and/or calculation may not attract full marks.
- Marks may not be awarded for carelessly arranged work.

Time Allowed

2 hours + 5 minutes reading time

Question 1 (12 marks) Start a NEW booklet. **Marks**

a) Calculate $\sqrt[3]{1{\cdot}35\times 0{\cdot}479}$ to 3 significant figures 1

b) Change $68{\cdot}69°$ to degrees, minutes and seconds 1

c) Find x if $\tan 20° = \cot\ (x+30)°$ 2

d) Express as a simplified surd:

(i) $\sqrt{192}$ 1

(ii) $\sqrt{147}-4\sqrt{3}$ 1

(iii) $4\sqrt{3}\times\sqrt{48}$ 1

(iv) $\dfrac{\sqrt{125}}{5}$ 1

e) If $\sin\theta = 0{\cdot}45$ and θ is acute, find θ to the nearest minute 1

f) Change $N40°W$ to a 3 digit bearing 1

g) If $a=4,\ b=-2$ *and* $c=-3$, find the value of $3(bc)^2-a+c$ 2

Question 2 (12 marks) Start a NEW booklet. **Marks**

a) Simplify, in exact form, the value of $\cos 30°+\sin 45°$, with a rational denominator 2

b) Factorise x^2+5x-6 1

c) Solve $4x+7=3(x-2)$ 2

Question 2 continued over/

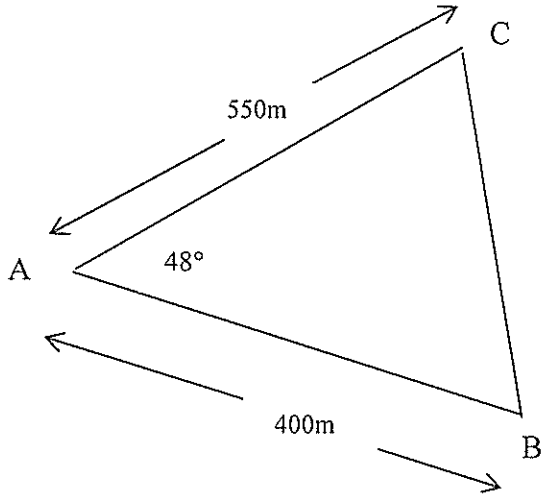
d) Solve:

(i) $3^{x-1} = 9^{2x}$ 2

(ii) $5-3x<7$ 1

(iii) $|2x+5|<3$ 2

e) In ΔABC, $AC = 550m$, $AB = 400m$ and $\angle CAB = 48°$. Calculate the length of BC, to the nearest metre. 2



NOT TO SCALE

Question 3 (12 marks) Start a NEW booklet. **Marks**

a) Express $0 \cdot \dot{2}\dot{3}$ as a fraction 2

b) If $\tan\theta = -\frac{2}{3}$ and $\sin\theta > 0$, find the exact value of:

(i) $\cot\theta$ 1

(ii) $\sin\theta$ 2

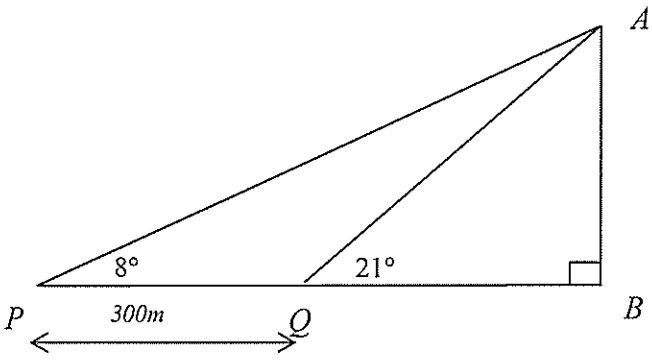
c) Solve $|x-1| = 4$ 2

d) Solve $\sin\theta + \sqrt{3}\cos\theta = 0$ for θ, where $0° \le \theta \le 360°$ 3

e) Simplify $1+\tan^2(90-\theta)$ 2

Question 4 (12 marks) Start a NEW booklet. **Marks**

a) From point P, Marc finds that the angle of elevation of the top A of a rock pillar AB is $8°$. After walking $300m$ directly towards the pillar to the point Q he finds that the angle of elevation of A is $21°$.



NOT TO SCALE

(i) Copy the diagram and find $\angle PAQ$. 1

(ii) Calculate the length of AQ. 2

(iii) Find the height of the rock pillar AB. 2

b) Simplify:

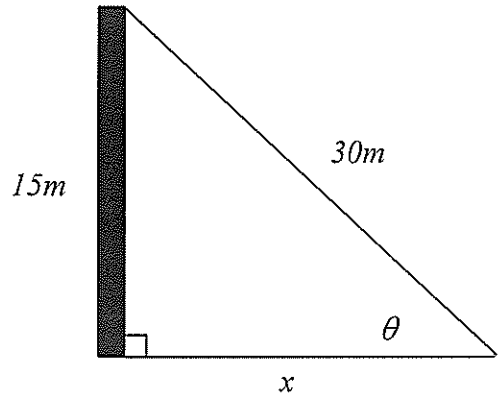
(i) $\dfrac{2x^2-3xy}{xy-y^2} \div \dfrac{4x-6y}{2x^2-2xy}$ 2

(ii) $\dfrac{a^3+b^3}{a^2-b^2}$ 2

(iii) $\dfrac{2}{m^2-4}-\dfrac{1}{m^2-3m+2}$ 3

Question 5 (12 marks) Start a NEW booklet. **Marks**

a) A vertical pole of height *15m* stands on the level ground and a straight wire *30m* long joins the top of the pole onto the ground.



NOT TO SCALE

Find:

(i) the distance x of this point from the foot of the pole (in exact form) 1

(ii) the angle the wire makes with the ground 2

b) Find the exact value of:

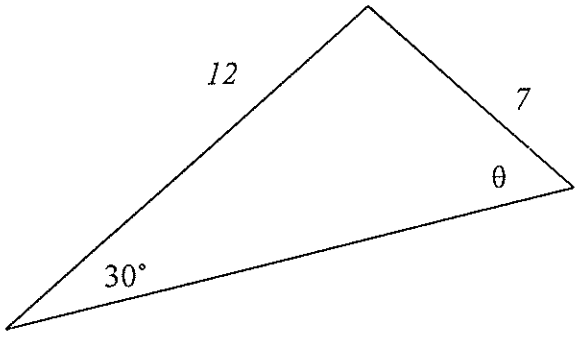
(i) $\sin 240°$ 1

(ii) $\cos(30)°$ 1

(iii) $\text{cosec}\, 45°$ 1

(iv) $\cot 420°$ 1

c) Find all possible value/s of θ, to the nearest degree.



NOT TO SCALE

3

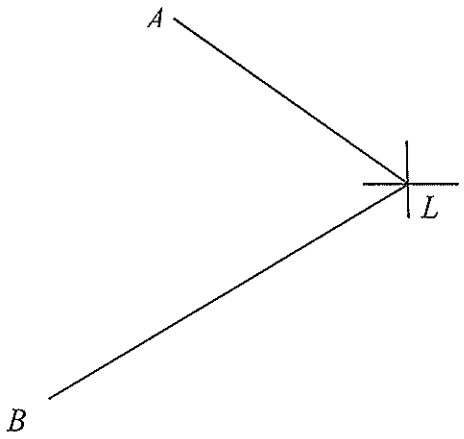
d) Express $\dfrac{\sqrt{2}-1}{2\sqrt{2}-1}$ in the form $a+b\sqrt{2}$ and state the values of a and b. 2

Question 6 (12 marks) Start a NEW booklet. **Marks**

a) Simplify $\sqrt{169a^6b^2}$ 1

b) A is $5km$ from a lighthouse on a bearing of $323°$. Also, B is 12km and at a bearing of $233°$ from the same lighthouse.

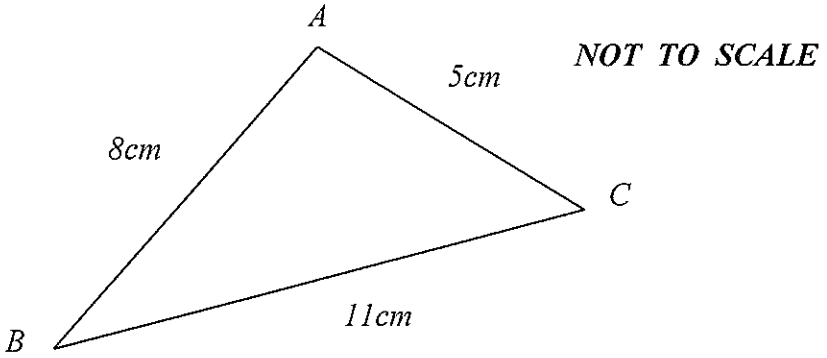
(i) Copy and complete the diagram showing the above information on your page, $\frac{1}{3}$ page size.



(ii) Calculate the distance of AB. 1

(iii) Find the bearing of B from A. 2

c) In ΔABC, $BC = 11cm$, $AC = 5cm$ *and* $AB = 8cm$.



(i) Calculate $\angle ABC$, to the nearest degree. 2

(ii) Find the area of ΔABC 2

d) Solve $2x^2 = x + 5$ by using the quadratic formula, leaving your answer in exact surd form. 2

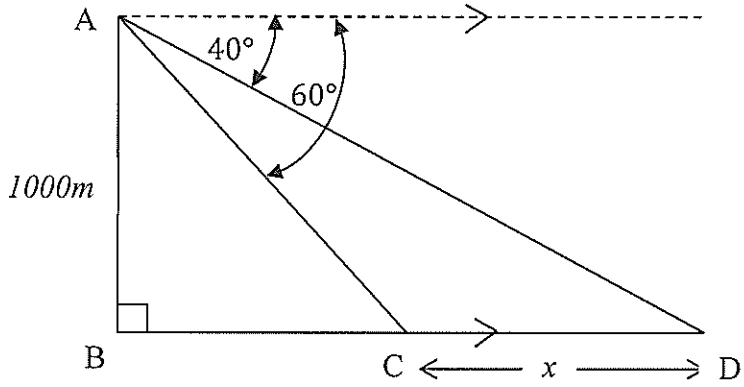
e) Sketch the graph $y = \cos\theta$ for $0° \le \theta \le 360°$, using a $\frac{1}{3}$ page size. 2

Question 7 (12 marks) Start a NEW booklet. **Marks**

a) Solve $x^2 - 10x - 1 = 0$ by completing the square, leaving your answer in exact form. 2

b) Solve simultaneously $\begin{cases} 3x - 2y = 14 \quad ----\boxed{1} \\ xy = 12 \quad ------\boxed{2} \end{cases}$ 3

c) From an aircraft 1000m above the ground, the angle of depression of the top of two houses (ignoring their height) in line with the aeroplane are $40°$ and $60°$ respectively. How far apart are the houses?



NOT TO SCALE

3

d) Prove $(1 - \tan x)^2 + (1 + \tan x)^2 = 2\sec^2 x$ 2

e) If $x = 5\sin\theta$ and $y = 5\cos\theta - 1$, eliminate θ to find an expression relating x and y. 2

End of Examination

Yr 11 (2U) Sem (1) Exam 2013

Q1

(a) $\sqrt[3]{1.35 \times 0.479} = 0.865$

(b) $68.69^\circ = 68^\circ 41' 24''$

(c) $\tan 20^\circ = \cot(x+30)^\circ$
$\tan 20^\circ = \cot(90-20)^\circ$
$= \cot 70^\circ$
$\therefore x + 30 = 70$
$x = 40^\circ$

(d) (i) $\sqrt{192} = \sqrt{64 \times 3}$
$= 8\sqrt{3}$

(ii) $\sqrt{147} - 4\sqrt{3} = \sqrt{49 \times 3} - 4\sqrt{3}$
$= 7\sqrt{3} - 4\sqrt{3}$
$= 3\sqrt{3}$

(iii) $4\sqrt{3} \times \sqrt{48} = 4\sqrt{3} \times \sqrt{16 \times 3}$
$= 4\sqrt{3} \times 4\sqrt{3}$
$= 48$

(iv) $\frac{\sqrt{125}}{5} = \frac{5\sqrt{5}}{5}$
$= \sqrt{5}$

(e) $\sin\theta = 0.45$
$\theta = 26^\circ 45'$

(f) $N40^\circ W = 320^\circ$

(g) $3(bc)^2 - a + c$
$= 3(6)^2 - 4 - 3$
$= 101$

Q2

(a) $\cos 30^\circ + \sin 45^\circ$
$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$
$= \frac{\sqrt{6}+2}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{\sqrt{12}+2\sqrt{2}}{4}$
$= \frac{\sqrt{3}+\sqrt{2}}{2}$

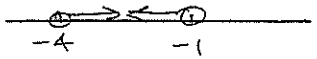
(b) $x^2 + 5x - 6 = (x+6)(x-1)$

(c) $4x + 7 = 3(x-2)$
$4x + 7 = 3x - 6$
$x = -13$

(d) (i) $3^{x-1} = 9^{2x}$
$3^{x-1} = 3^{2(2x)}$
$x - 1 = 4x$
$-3x = 1$
$x = -1/3$

(ii) $5 - 3x < 7$
$-3x < 2$
$x > -2/3$

(iii) $|2x+5| < 3$
$2x + 5 < 3$ OR $-(2x+5) < 3$
$2x < -2$ $-2x - 5 < 3$
$x < -1$ $-2x < 8$
$x > -4$



$\therefore -4 < x < -1$

(e) $BC^2 = 550^2 + 400^2 - 2(550)(400)\cos 48^\circ$
$= 168082.53$
$BC = \sqrt{168082.53}$
$BC = 410m$

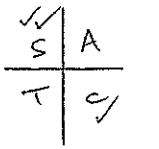
Q3

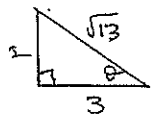
(a) $0.\dot{2}\dot{3}$
let $x = 0.2323\ldots$
$100x = 23.2323\ldots$
$99x = 23$
$x = \frac{23}{99}$

Q3 cont

(b) (i) $\tan\theta = -\frac{2}{3}$, $\sin\theta > 0$
$\cot\theta = -\frac{3}{2}$

(ii) $\sin\theta = \frac{2}{\sqrt{13}}$

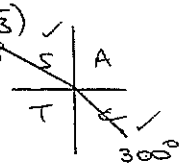




(c) $|x-1| = 4$
$x - 1 = 4$ OR $-(x-1) = 4$
$x = 5$ $-x + 1 = 4$
$-x = 3$
$x = -3$

$\therefore x = 5, -3$

(d) $\sin\theta + \sqrt{3}\cos\theta = 0$, $0^\circ \le \theta \le 360^\circ$
$\sin\theta = -\sqrt{3}\cos\theta$
$\frac{\sin\theta}{\cos\theta} = -\sqrt{3}$
$\tan\theta = -\sqrt{3}$.
$\theta = -\tan^{-1}(\sqrt{3})$
$= -60^\circ$

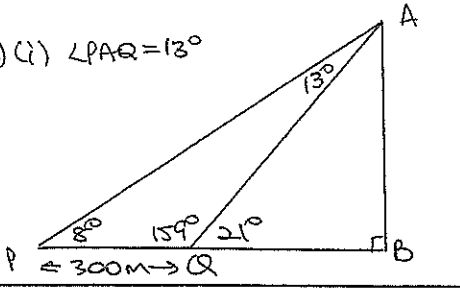


$\therefore \theta = 120^\circ, 300^\circ$

(e) $1 + \tan^2(90-\theta) = 1 + \cot^2\theta$
$= \operatorname{cosec}^2\theta$

Q4

(a) (i) $\angle PAQ = 13^\circ$



(ii) $\frac{AQ}{\sin 8^\circ} = \frac{300}{\sin 13^\circ}$
$AQ = \frac{300\sin 8^\circ}{\sin 13^\circ}$
$= 185.6m$ (1 dp)

(iii) $\sin 21^\circ = \frac{AB}{AQ}$
$\sin 21^\circ = \frac{AB}{185.6}$
$AB = 185.6\sin 21^\circ$
$= 66.5m$ (1 dp)

(b) (i) $\frac{2x^2 - 3xy}{xy - y^2} \div \frac{4x - 6y}{2x^2 - 2xy}$
$= \frac{x(2x-3y)}{y(x-y)} \times \frac{2x(x-y)}{2(2x-3y)}$
$= \frac{x^2}{y}$

(ii) $\frac{a^3+b^3}{a^2-b^2} = \frac{(a+b)(a^2-ab+b^2)}{(a+b)(a-b)}$
$= \frac{a^2-ab+b^2}{a-b}$

(iii) $\frac{2}{m^2-4} - \frac{1}{m^2+3m+2}$
$= \frac{2}{(m-2)(m+2)} - \frac{1}{(m-2)(m-1)}$
$= \frac{2(m-1) - (m+2)}{(m-2)(m+2)(m-1)}$
$= \frac{2m-2-m-2}{(m-2)(m+2)(m-1)}$
$= \frac{m-4}{(m-2)(m+2)(m-1)}$

Q5

(a)(i) $x^2+15^2=30^2$

$x^2=675$

$x=\sqrt{675}$

$x=\sqrt{5\cdot5\times3}$

$=15\sqrt{3}$

(ii) $\sin\theta=\frac{15}{30}$

$=\frac{1}{2}$

$\therefore\theta=30^\circ$

(b)(i) $\sin240^\circ=-\frac{\sqrt{3}}{2}$

(ii) $\cos30^\circ=\frac{\sqrt{3}}{2}$

(iii) $\operatorname{cosec}45^\circ=\frac{1}{\sin45^\circ}$

$=\frac{1}{1/\sqrt{2}}$

$=\sqrt{2}$

(iv) $\cot420^\circ=\cot60^\circ$

$=\frac{1}{\tan60^\circ}$

$=\frac{1}{\sqrt{3}}$

(c) $\frac{\sin\theta}{12}=\frac{\sin30^\circ}{7}$

$\sin\theta=\frac{12\sin30^\circ}{7}$

$\theta=59^\circ,\ 180-59^\circ$

$\therefore\theta=59^\circ,\ 121^\circ$

(note that $121^\circ+30^\circ=151^\circ<180^\circ$

$\therefore 121^\circ$ is also valid

(d) $\frac{\sqrt{2}-1}{2\sqrt{2}-1}\times\frac{2\sqrt{2}+1}{2\sqrt{2}+1}$

$=\frac{4+\sqrt{2}-2\sqrt{2}-1}{8-1}$

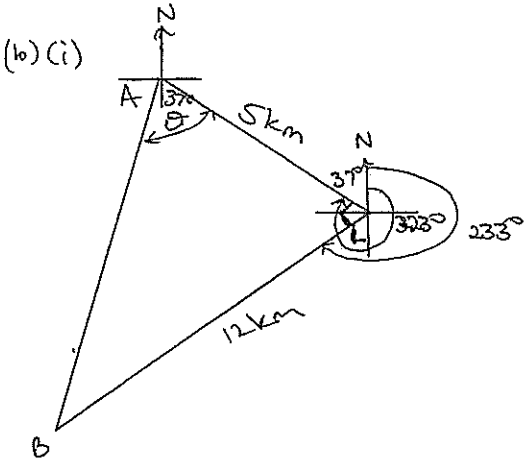
$=\frac{3-\sqrt{2}}{7}$

$\therefore a=3/7,\ b=-1/7$

Q6

(a) $\sqrt{169a^6b^2}=13a^3b$

(b)(i)



(ii) $AB^2=5^2+12^2$

$=169$

$AB=13km$

(iii) $\tan\theta=\frac{12}{13}$

$\theta=\tan^{-1}(12/13)$

$=67^\circ23'$

$\therefore$ bearing is $180^\circ+30^\circ23'$

$=210^\circ23'$

Q6 cont

(c)(i) $5^2=8^2+11^2-2(8)(11)\cos\angle ABC$

$25=185-176\cos\angle ABC$

$-160=-176\cos\angle ABC$

$\cos\angle ABC=\frac{10}{11}$

$\angle ABC=\cos^{-1}(\frac{10}{11})$

$=25^\circ$

(ii) $A=\frac{1}{2}\times8\times11\sin25^\circ$

$=18.6cm^2$ (1 d.p)

(d) $2x^2=x+5$

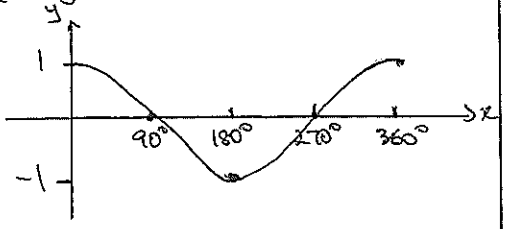
$2x^2-x-5=0$

$x=\frac{1\pm\sqrt{(-1)^2-4(2)(-5)}}{2(2)}$

$=\frac{1\pm\sqrt{1+40}}{4}$

$=\frac{1\pm\sqrt{41}}{4}$

(e) $y=\cos\theta$



Q7

(a) $x^2-10x-1=0$.

$x^2-10x=1$

$x^2-10x+25=1+25$

$(x-5)^2=26$

$x-5=\pm\sqrt{26}$

$x=5\pm\sqrt{26}$

(b) $3x-2y=14$ ... ①

$xy=12$ ... ②

From ① $y=\frac{12}{x}$ ... ③

sub ③ into ①.

$3x-2\left(\frac{12}{x}\right)=14$

$3x^2-24=14x$

$3x^2-14x-24=0$.

$\frac{(3x-18)(3x+4)}{3}=0$

$\frac{3(x-6)(3x+4)}{3}=0$

$(x-6)(3x+4)=0$.

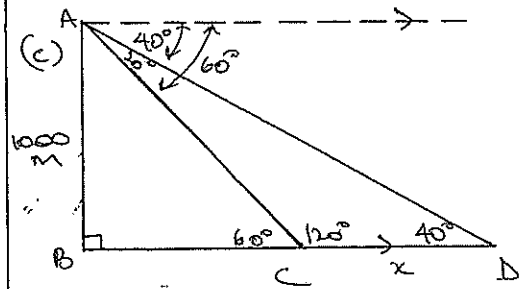
$x=6,\ -4/3$.

when $x=6$, $y=2$

" $x=-4/3$, $y=-9$.

$\therefore(6,2)$ & $(-\frac{4}{3},-9)$

(c)



$\sin60^\circ=\frac{1000}{AC}$

$AC=\frac{1000}{\sin60^\circ}$

$=1154.7m$ (1 d.p)

$\frac{x}{\sin20^\circ}=\frac{1154.7}{\sin40^\circ}$

$x=\frac{1154.7\sin20^\circ}{\sin40^\circ}$

$=614.4m$ (1 d.p)

Q7 cont

(d) $(1-\tan x)^2 + (1+\tan x)^2 = 2\sec^2 x$

LHS

$(1-\tan x)^2 + (1+\tan x)^2$

$= 1 - 2\tan x + \tan^2 x + 1 + 2\tan x + \tan^2 x$

$= 2 + 2\tan^2 x$

$= 2(1+\tan^2 x)$

$= 2\sec^2 x$

$=$ RHS.

(e) $x = 5\sin\theta$

$\sin\theta = \frac{x}{5}$

$\sin^2\theta = \left(\frac{x}{5}\right)^2$

Also, $y = 5\cos\theta - 1$

$5\cos\theta = y+1$

$\cos\theta = \frac{y+1}{5}$

$\cos^2\theta = \left(\frac{y+1}{5}\right)^2$

Now, $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y+1}{5}\right)^2 = 1$

OR

$x^2 + (y+1)^2 = 25.$